

Sheet #1

1. Determine the order N of the *Butterworth filter* for which $A_{\max} = 1\text{dB}$, $A_{\min} \geq 20\text{dB}$, and the selectivity ratio $\omega_s/\omega_p = 1.3$. What is the actual value of minimum stopband attenuation realized? If $A_{\min}$ is to be exactly 20dB , to what value can $A_{\max}$ be reduced?
2. Calculate the value of attenuation obtained at frequency 1.6 times the 3-dB frequency of a seventh order *Butterworth filter*.
 $N=7$ $\omega_s = 1.6 \omega_p$ $A_{\max}$
3. Find the natural modes of a *Butterworth filter* with a 1-dB bandwidth of 10^3 rad/sec and $N=5$.
 $\omega_p = 10^3$ $A_{\max}$
4. Design a *Butterworth filter* that meets the following low-pass specifications: $f_p = 10\text{KHz}$, $A_{\max} = 2\text{dB}$, $f_s = 15\text{KHz}$, and $A_{\min} = 15\text{dB}$. Find N , the natural modes and $T(s)$. What is the attenuation at 20KHz ?
5. Sketch $|T|$ for a seventh-order low-pass *Chebyshev filter* with $\omega_p = 1$ rad/sec and $A_{\max} = 1\text{dB}$. Determine the values of ω at which $|T| = 1$ and the values of ω at which $|T| = 1/\sqrt{1+\epsilon^2}$. Indicate these values on your sketch. Determine $|T|$ at $\omega = 2$ rad/sec, and indicate this point on your sketch. For large values of ω , at what rate (in dB/octave) does the transmission decrease?
القانون $\omega = \omega_p \cos \theta$ $\omega = \omega_p \cos 2\theta$
6. Contrast the attenuation provided by the a fifth-order *Chebyshev filter* at $\omega_s = 2\omega_p$ to that provided by a *Butterworth filter* with the same order. For both $A_{\max} = 1\text{dB}$. Sketch $|T|$ for both filters at the same axes.
7. It is required to design a low-pass filter to meet the following specifications: $f_p = 3.4\text{KHz}$, $A_{\max} = 1\text{dB}$, $f_s = 4\text{KHz}$, and $A_{\min} = 35\text{dB}$. Find the required order of *Chebyshev filter*. What is the excess (above 35dB) stopband attenuation obtained? Also find the poles and transfer function.
8. Design a first-order opamp-RC low-pass filter having 3-dB frequency of 10KHz , a dc gain magnitude of 10, and an input resistance of $10\text{K}\Omega$.
9. Design a first-order opamp-RC high-pass filter having 3-dB frequency of 100Hz , a high frequency gain magnitude of unity, and an high frequency input resistance of $100\text{K}\Omega$.
10. Design a first-order opamp-RC spectrum shaping network with transmission zero frequency of 1KHz , a pole frequency of 100KHz , and a dc gain magnitude of unity. The low frequency input resistance is to be $1\text{K}\Omega$. What is the high-frequency gain that results? Sketch the magnitude of the transfer function versus frequency.

11. By cascading a first-order opamp-RC low-pass circuit with a first-order opamp-RC high-pass circuit, one can design a wideband bandpass filters. Provide such a design for the case of the midband gain is 12dB and the 3-dB bandwidth extends from 100Hz to 10KHz. Select appropriate component values under the constraint that no resistance higher than 100K Ω are to be used, and the input resistance are high as possible.
12. Derive $\angle(s)$ for the opamp-RC first order all pass filter. We wish to use this circuit as a variable phase shifter by adjusting R. If the input signal frequency is 10^4 rad/sec and if $C=10$ nF, find the values of R required to obtain phase shifts of -30° , -60° , -90° , -120° , and -150° .
13. Show that by interchanging R and C in the circuit of prob.12, the resulting phase shifts covers the range 0° to 180° (with 0° at high frequencies and 180° at low frequencies).

sheet #1

Butterworth and chebyshev filter

1) Given:

$$A_{max} = 1 \text{ dB} , A_{min} \geq 20 \text{ dB} , \text{ and } \frac{\omega_s}{\omega_p} = 1.3$$

Find

- order n of butterworth filter.
- what is actual value of min stop band attenuation
- if A_{min} is to be exactly 20 dB what is A_{max} ?

solution

$$A(\omega_s) \geq A_{min} \quad \text{في } \omega_s \text{ نريد } A_{min}$$

$$\text{and } A(\omega_s) = 10 \log \left[1 + \epsilon^2 \left(\frac{\omega_s}{\omega_p} \right)^{2n} \right]$$

$$\epsilon \quad \text{في } \omega_p \text{ نريد } A_{max}$$

$$\epsilon = \sqrt{10^{\frac{A_{max}}{10}} - 1} = \sqrt{10^{\frac{1}{10}} - 1} = 0.5088$$

في ω_p نريد A_{max}
في ω_s نريد A_{min}

$$A(\omega_s) = 10 \log \left[1 + (0.5088)^2 (1.3)^{2n} \right]$$

في ω_s نريد A_{min} في ω_p نريد A_{max}

$$n = 12$$

b) what is actual value of $A(\omega_s)$

يتم التعرف على القيمة $A(\omega_s)$ من خلال $12 = 20 \log A(\omega_s)$

$$A(\omega_s) = 10 \log [1 + (0.5088)^2 (1.3)^{2 \times 12}]$$

$$A(\omega_s) = 21.508 \text{ dB}$$

c) if $A_{\min} = 20 \text{ dB}$ what is $A_{\max}$

$$A_{\max} = 20 \log \sqrt{1 + \epsilon^2}$$

لا بد من التعرف على ϵ

$$A(\omega_s) = 10 \log [1 + \epsilon^2 (\frac{\omega_s}{\omega_p})^{2n}]$$

$$\therefore A(\omega_s) = A_{\min}$$

$$20 \text{ dB} = 10 \log [1 + \epsilon^2 (1.3)^{24}]$$

$$\frac{20}{10} = \log [1 + \epsilon^2 (1.3)^{24}]$$

$$\epsilon^2 = \frac{10^{\frac{20}{10}} - 1}{(1.3)^{24}} = 0.1823$$

$$A_{max} = 0.73 \text{ dB}$$

$$w_A = 1.6 \text{ w}_p \quad \sigma = 7 \text{ } \sigma_{\text{max}} = 30 \text{ dB}$$
 $A(\omega)$

لا قیام و استقامت
و لکن تکرار از Pass Band

solution

$$A(\omega) = 10 \log \left[1 + \epsilon^2 \left(\frac{\omega}{\omega_p} \right)^{2N} \right]$$

$\in \text{line segment } xy$

$$E = \sqrt{\frac{A_{\max}}{10} - 1} = \sqrt{\frac{3}{10} - 1} = 0.998$$

$$A_{\omega} = 101.9 [1 + (0.998)^2 (1.6)^{2 \times 7}]$$

3) Find natural mode of Butterworth filter with 1 dB band width 10^3 rad/s $N=5$

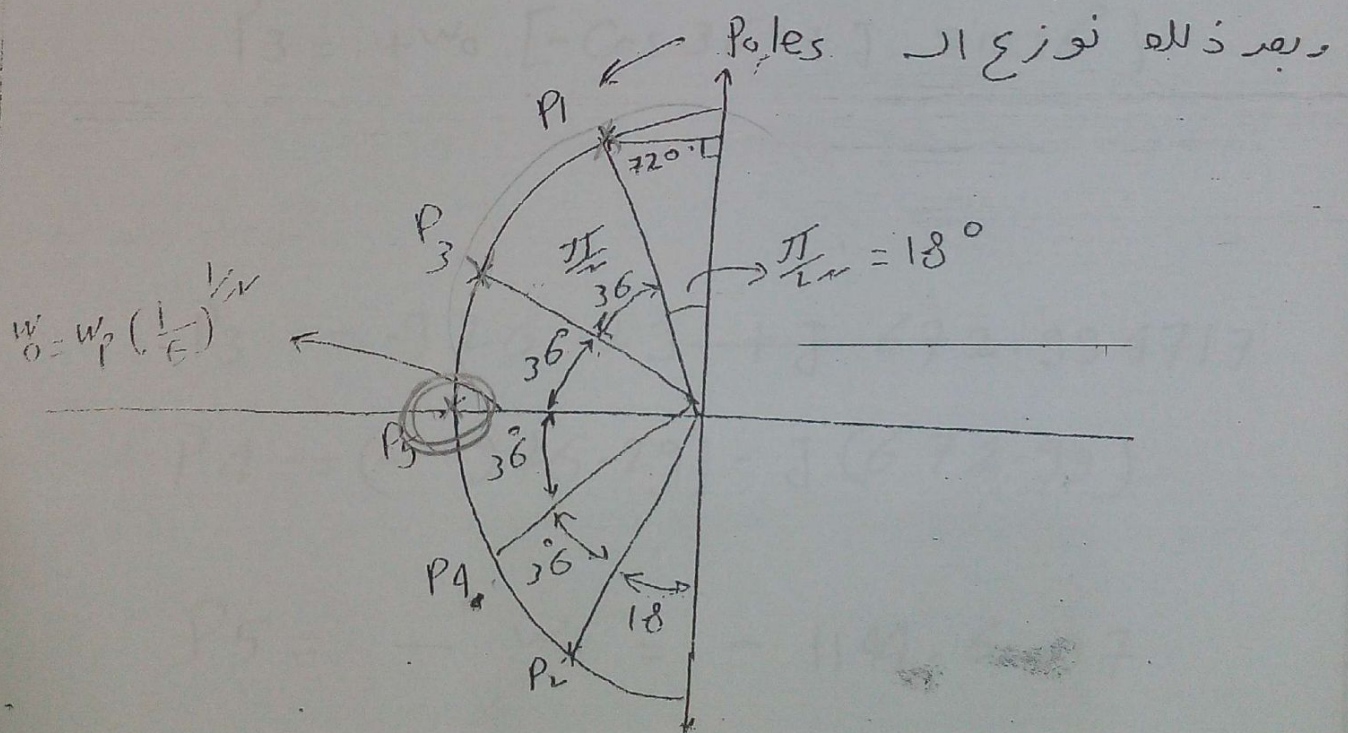
Solution:

$$A_{\max} = 1 \text{ dB}, \quad \omega_p = 10^3 \text{ rad/s}, \quad N = 5$$

أولاً نوجد ϵ

$$\epsilon = \sqrt{\frac{A_{\max}}{10} - 1} = \sqrt{\frac{1}{10} - 1}$$

$$\epsilon = 0.5088$$



$$\omega_0 = 10^3 \times \left(\frac{1}{0.5088}\right)^{1/5} = 1144.697$$

$$P_1 = 0 + j\omega$$

$$= -\omega_0 \cos 72^\circ + j\omega_0 \sin 72^\circ$$

$$= \omega_0 [-\cos 72^\circ + j \sin 72^\circ]$$

$$P_1 = -353 + j 1088.67$$

✓ ω_0
rad/s

$$P_2 = -353 - j 1088.67$$

$$P_3 = +\omega_0 [-\cos 36^\circ + j \sin 36^\circ]$$

$$P_3 = -926.29 + j 672.994717$$

$$P_4 = (-926.29) - j (672.99)$$

$$P_5 = -\omega_0 = -1144.637$$

4) Design butter worth for

$$f_p = 10 \text{ KHz}, A_{\max} = 2 \text{ dB}, f_s = 15 \text{ KHz}, A_{\min} = 50 \text{ dB}$$

- find n
- Natural mode
- $T(\omega)$
- what is the attenuation provided at 20 KHz ?

Solution:-

الآن نقوم بحساب ϵ

$$\epsilon = \sqrt{10^{\frac{A_{\max}}{10}} - 1} = \sqrt{10^{\frac{2}{10}} - 1} = 0.765$$

To find n

$$A(\omega_s) = 10 \log \left[1 + (\epsilon \cdot 0.765)^2 \left[\frac{15}{10} \right]^{2 \times n} \right]$$

وعند ω_s نريد أن نحصل على $A(\omega_s) = 50 \text{ dB}$ لأن $A_{\min}$ هو الحد الأدنى للضعف المطلوب عند ω_s

$$n = \frac{\log \left[\left(10^{\frac{A(\omega_s)}{10}} - 1 \right) / \epsilon^2 \right]}{2 \log \left(\frac{\omega_s}{\omega_p} \right)}$$

$$= 4.88$$

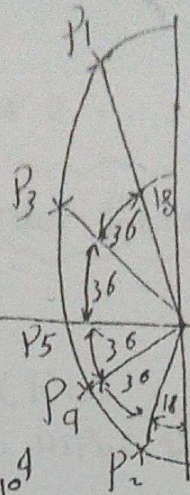
$$n = 5 = \text{العدد الطبيعي الأكبر من } 4.88$$

natural mode

$$\omega_0 = \omega_p \left(\frac{1}{\epsilon} \right)^{1/2}$$

$$= 2\pi \times f_p \left(\frac{1}{\epsilon} \right)^{1/2}$$

$$= 2\pi \times 10 \times 10^3 \left[\frac{1}{0.765} \right]^{1/2} = 6.63 \times 10^4$$



$$P_{1,2} = -\omega_0 \cos 72^\circ \pm j \omega_0 \sin 72^\circ$$

$$P_{1,2} = -0.3089\omega_0 \pm j 0.951\omega_0$$

$$P_{3,4} = -\omega_0 \cos 36^\circ \pm j \omega_0 \sin 36^\circ$$

$$P_{3,4} = -\omega_0 \times 0.809 \pm j 0.588 \omega_0$$

$$P_{3,4} = -0.809\omega_0 \pm j 0.588\omega_0$$

$$P_5 = -\omega_0$$

$$(s - P_1)(s - P_2) = s^2 + 2\zeta\omega_0 s + \omega_0^2$$

Conjugated real amplitude of poles

av. vel. temp.

sheet * 2

First order filter

8

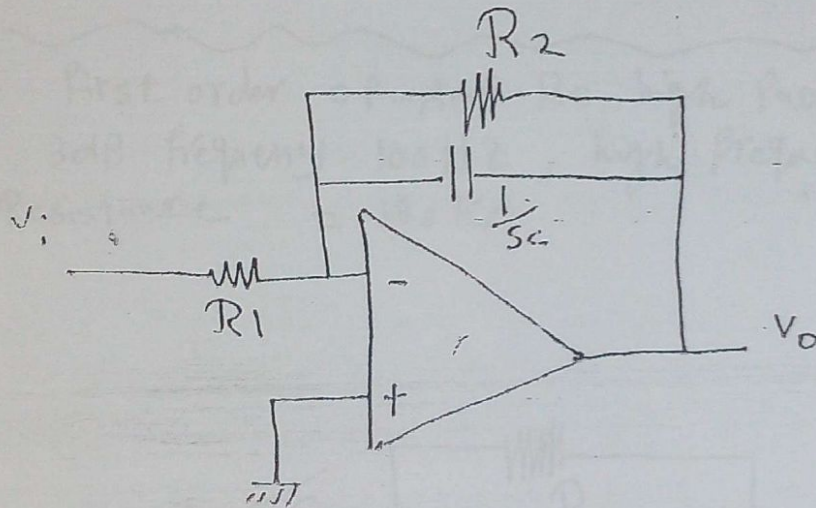
Q1) Given

First order LPF (RC), $f_0 = 10 \text{ kHz}$

dc gain = 10, input Resistance = $10 \text{ k}\Omega$

Solution

LPF



$$R_{in} = R_1 = 10 \text{ k}\Omega$$

$$\omega_0 = \frac{1}{CR_2}$$

$$|dc \text{ gain}| = \frac{R_2}{R_1}$$

$$10 = \frac{R_2}{R_1}$$

$$\therefore R_2 = 10 R_1$$

$$\therefore R_2 = 100 \text{ k}\Omega$$

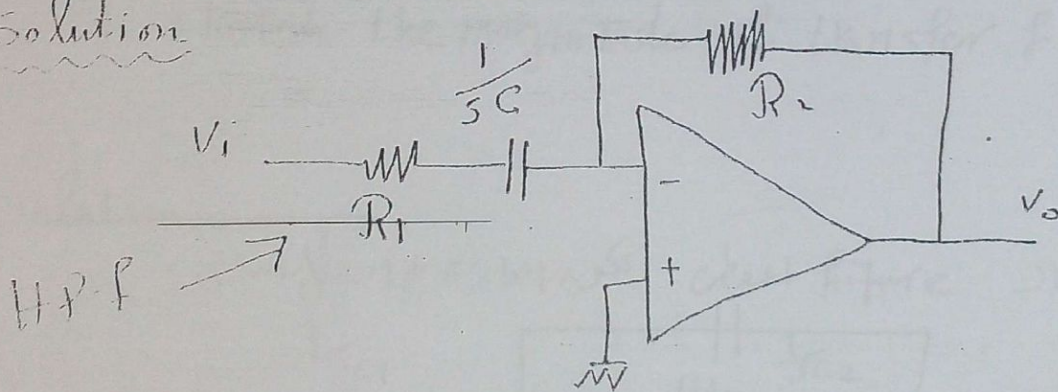
$$\omega_0 = 2\pi f_0 = \frac{1}{C R_2}$$

$$\therefore C = \frac{1}{2\pi f_0 R_2} = \frac{1}{2\pi \times 10 \times 10^3 \times 100 \times 10^3}$$

$$C = 0.159 \text{ nF}$$

2) Design first order op-Amp RC high Pass filter with 3dB frequency 100 Hz, high frequency gain = 1, high frequency input Resistance = 100 k Ω .

Solution



high frequency input Resistance = $R_1 = 100 \text{ k}\Omega$

$$\therefore \left| \text{gain} \right| = \frac{R_2}{R_1} = 1$$

$$\therefore R_1 = R_2 = 100 \text{ k}\Omega$$

$$\omega_0 = \frac{1}{C R_1}$$

$$C = \frac{1}{2\pi f_0 R_1} = \frac{1}{2\pi \times 100 \times 100 \times 10^3}$$

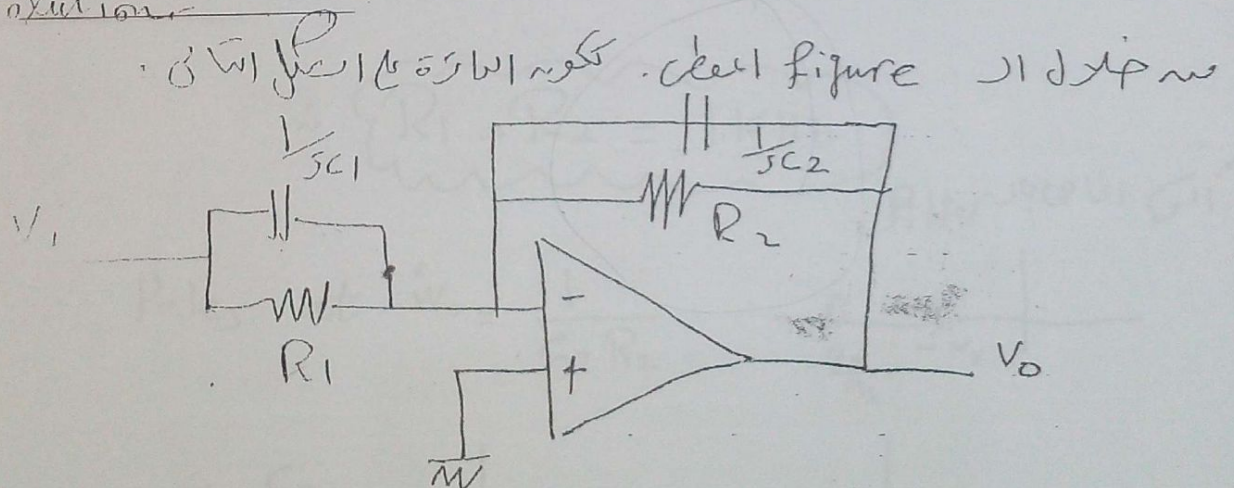
$$= 15.9 \text{ nF}$$

210) a) design first order op-amp RC, spectrum shaping network with zero frequency of 1 kHz pole frequency 100 kHz & dc gain = 1 & the lower frequency resistance to be 1 k Ω

b) what is high frequency gain?

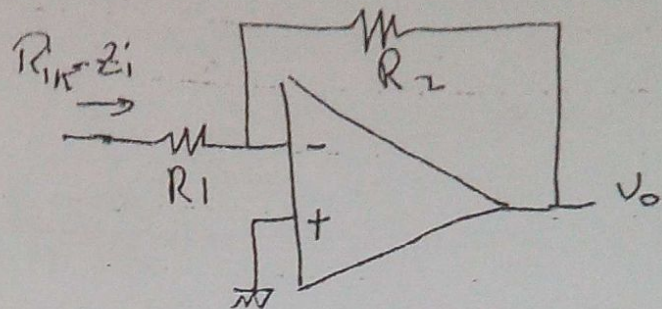
c) sketch the magnitude of transfer fn?

Solution:



yes, Low Frequency Resistance to 0 & 100k Ω so that any capacitance be " ∞ "

$$R_{in} = R_1$$



$$R_{in} = R_1 = 1k\Omega$$

and zero's at 1KHz

Poles at 100KHz

$$dc \text{ gain} = 1$$

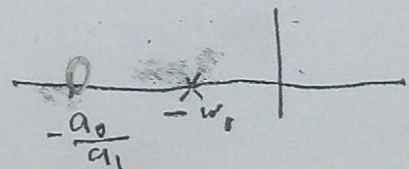
$$\therefore dc \text{ gain} = -\frac{R_2}{R_1}$$

$$\therefore |dc \text{ gain}| = \frac{R_2}{R_1}$$

$$\therefore \{ R_1 = R_2 = 1k\Omega \}$$

فیلٹر ایف ایچ

Poles at $\omega_0 = \frac{1}{C_2 R_2}$



$$\therefore C_2 = \frac{1}{2\pi f_0 R_2} = \frac{1}{2\pi \times (100k\text{Hz}) \times 1 \times 10^3}$$

$\uparrow$
 100kHz

$$C_2 = 1.59 \text{ nF}$$

$$\text{from } |T(s)| = \frac{a_1 s + a_0}{s + \omega_0}$$

نقطه قطب را ω_0 و Zero's را s می‌نویسند

$$s = \frac{a_1}{a_0}$$

همانطور که می‌بینیم

$$\frac{a_1}{a_0} = \frac{1}{C_1 R_1}$$

فرکانس زاویه‌ای

$$2\pi \times 1 \times 10^3 = \frac{1}{C_1 R_1}$$

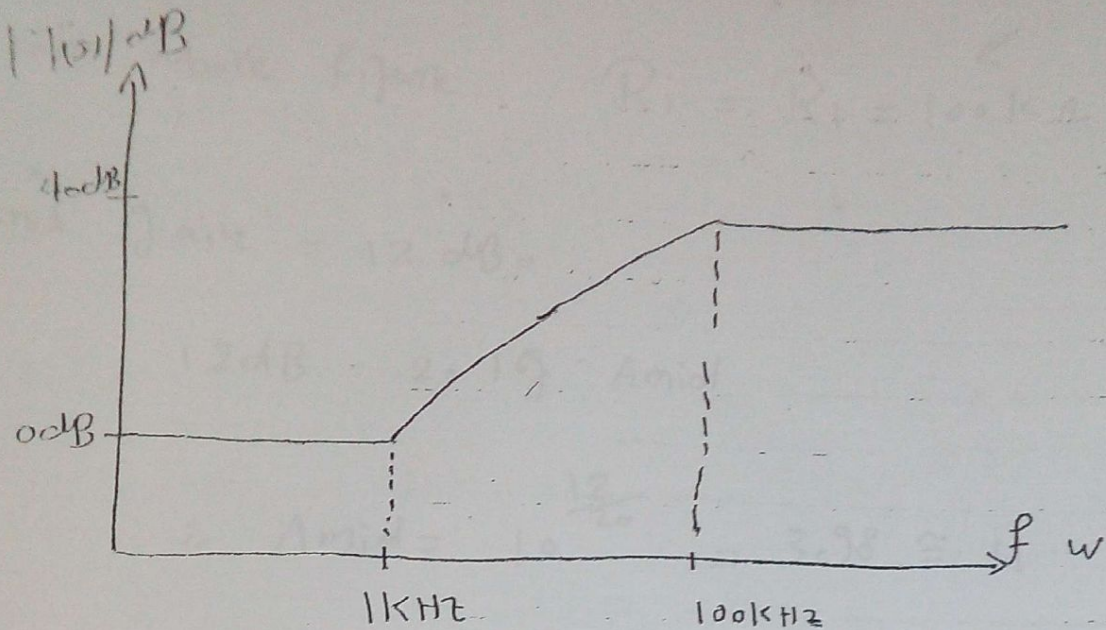
$$\therefore C_1 = \frac{1}{2\pi \times 1 \times 10^3 \times 1 \text{ k}\Omega} = 159 \text{ nF}$$

$$b) \text{ High frequency gain} = -\frac{C_1}{C_2} = \frac{159 \text{ nF}}{1.59 \text{ nF}}$$

$$= -100 = -40 \text{ dB}$$

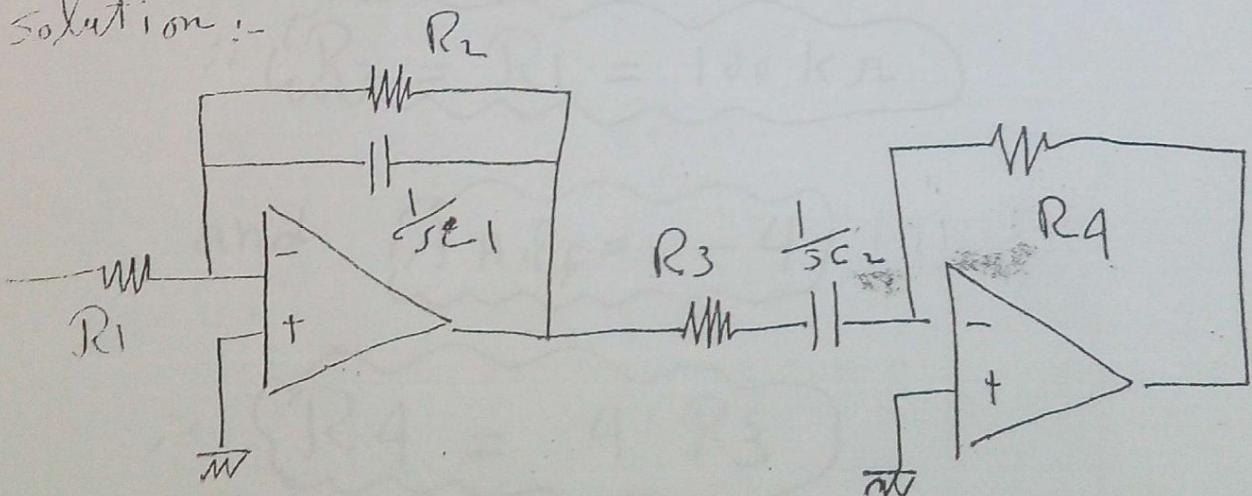
$$\therefore \text{dc gain} = 1 = 0 \text{ dB}$$

$|T(s)|$ را بکش



11) by Cascade a first op-Amp L.P.F with first order ~~low~~ H.P.F. To obtain wide band pass filter provide such a design for the case the mid band gain 12 dB and 3 dB band width extends from 100 Hz to 10 kHz, select appropriate component value under that no Resistance higher than 100 kHz, and i/p Resistance is to be high as possible?

Solution :-



from above figure

$$R_i = R_1 = 100 \text{ k}\Omega$$

and gain = 12 dB =

$$12 \text{ dB} = 20 \log A_{mid}$$

$$\therefore A_{mid} = 10^{\frac{12}{20}} = 3.98 \approx 4$$

$$\text{Total gain} = 4 = A_{L.P.F} \times A_{H.P.F}$$

$$\therefore A_{L.P.F} = -\frac{R_2}{R_1}$$

At

$$A_{H.P.F} = -\frac{R_4}{R_3}$$

$$\therefore R_2 = -A_{L.P.F} \times R_1$$

معنا نفوم بمل نرفغانى لالة ويكس -1

$$\therefore R_2 = R_1 = 100 \text{ k}\Omega$$

$$\text{and } A_{H.P.F} = -4$$

$$\therefore R_4 = 4 R_3$$

and make

$$R_4 = 100 \text{ k}\Omega$$

$$R_3 = 25 \text{ k}\Omega$$

$$\therefore f_{0 \text{ L.P.F}} = 10 \text{ kHz}$$

$$\omega_{0 \text{ L.P.F}} = \frac{1}{C_1 R_1}$$

$$\therefore C_1 = \frac{1}{2\pi f_{0 \text{ L.P.F}} R_1} = \frac{1}{2\pi \times 10 \times 10^3 \times 100 \times 10^3}$$

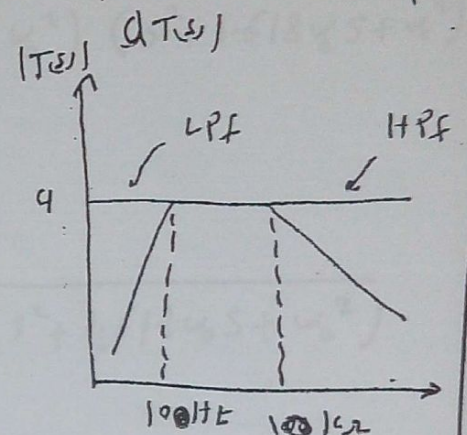
$$C_1 = 0.159 \text{ nF}$$

For high pass filter $f_{0 \text{ H.P.F}} = 100 \text{ Hz}$

$$\therefore \omega_{0 \text{ H.P.F}} = \frac{1}{C_2 R_3}$$

$$\therefore C_2 = \frac{1}{2\pi \times 100 \text{ Hz} \times 25 \times 10^3} = 63.7 \text{ nF}$$

Amplitude response of the filter



dc gain
↓

8

$$T(s) = \frac{K \omega_0^5}{(s-p_1)(s-p_2)(s-p_3)(s-p_4)(s-p_5)}$$

$$T(s) = \frac{\omega_0^5}{(s+\omega_0)(s^2+0.618\omega_0s+\omega_0^2)(s^2+1.618\omega_0s+\omega_0^2)}$$

$$T(s) = \frac{\omega_0^5}{(s+\omega_0)(s^2+0.618\omega_0s+\omega_0^2)(s^2+1.618\omega_0s+\omega_0^2)}$$

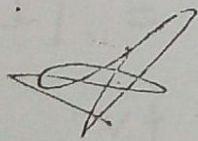
Find $|T(s)|$ at 20kHz

$20\text{kHz} > \omega_p$
attenuation exp. 5

$$\therefore A(\omega_s) = 10 \log \left[1 + \epsilon^2 \left(\frac{\omega_s}{\omega_p} \right)^{2n} \right]$$

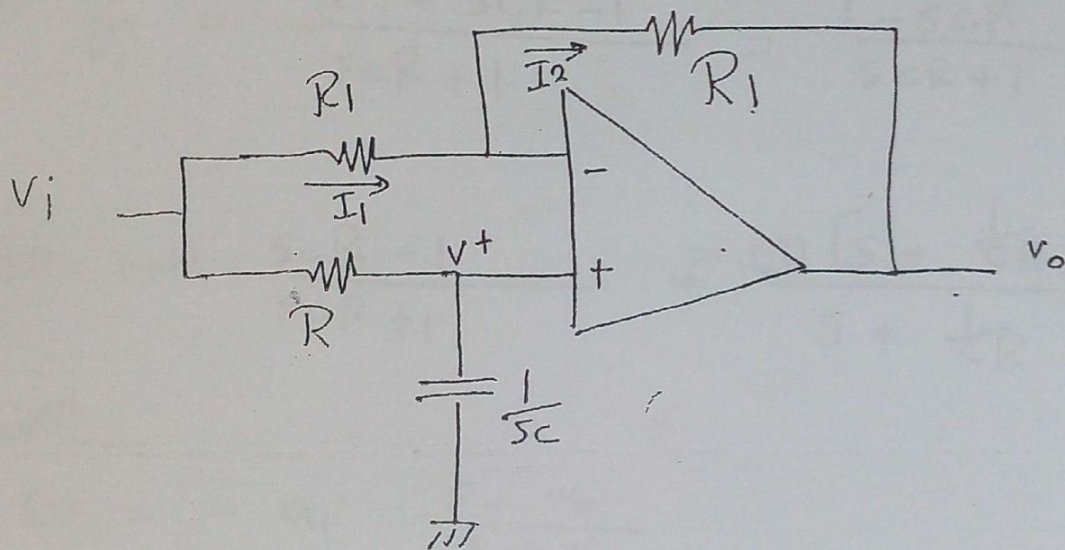
0.765 10kHz $2n$ 5

$$A(\omega_s) = 27.8 \text{ dB}$$



12) Derive $T(s)$ for the following (we wish to use circuit as phase shifter by adjusting R , if i/p signal frequency 10^4 rad/sec and $C = 10\text{ nF}$. Find the value R required to obtain phase shifts of $-30^\circ, -60^\circ, -90^\circ, -120^\circ$ & -150° .

Solution



$$T(s) = \frac{V_o}{V_i} \quad \text{---} \quad V^+ = V_i \cdot \frac{\frac{1}{sC}}{\frac{1}{sC} + R}$$

$$\therefore V^+ = V_i \cdot \frac{1}{1 + sCR}$$

$$\therefore I_1 = I_2$$

$$\frac{V_i - V^+}{R_1} = \frac{V^+ - V_o}{R_1}$$

$$V_i - V_i \frac{1}{sCR+1} = V_i \frac{1}{sCR+1} - V_o$$

$$V_o = V_i \left[\frac{1}{sCR+1} + \frac{1}{sCR+1} \cdot -1 \right]$$

$$T(s) = \frac{V_o}{V_i} = \frac{2 - sCR - 1}{sCR + 1} = \frac{1 - sCR}{sCR + 1}$$

$$T(s) = (-1) \frac{sCR - 1}{sCR + 1} = \frac{(-1) \left[s - \frac{1}{CR} \right]}{s + \frac{1}{CR}}$$

comparing

$$T(s) = -a_1 \frac{s - \omega_0}{s + \omega_0}$$

$$\therefore a_1 = +1 \quad \omega_0 = \frac{1}{CR}$$

To find Phase

$$T(s) = \frac{1 - sCR}{sCR + 1} \quad \because s = j\omega$$

$$T(s) = \frac{1 - j\omega CR}{j\omega CR + 1} = \frac{(-1) [j\omega CR - 1]}{j\omega CR + 1}$$

$$\phi = \frac{\tan^{-1} - \frac{wCR}{1}}{\tan^{-1} \frac{wCR}{1}}$$

المزاد

$$\tan^{-1} - \phi = -\tan^{-1} \phi$$

$$\phi = \frac{-\tan^{-1} wCR}{\tan^{-1} wCR}$$

$$\phi = -\tan^{-1} wCR - \tan^{-1} wCR$$

$$\phi = -2 \tan^{-1} wCR$$

$$\frac{-\phi}{2} = \tan^{-1} wCR$$

$$wCR = \tan \left(-\frac{\phi}{2} \right)$$

$$R = \frac{\tan \left(-\frac{\phi}{2} \right)}{wC}$$

10^4 rad/s 10 nF

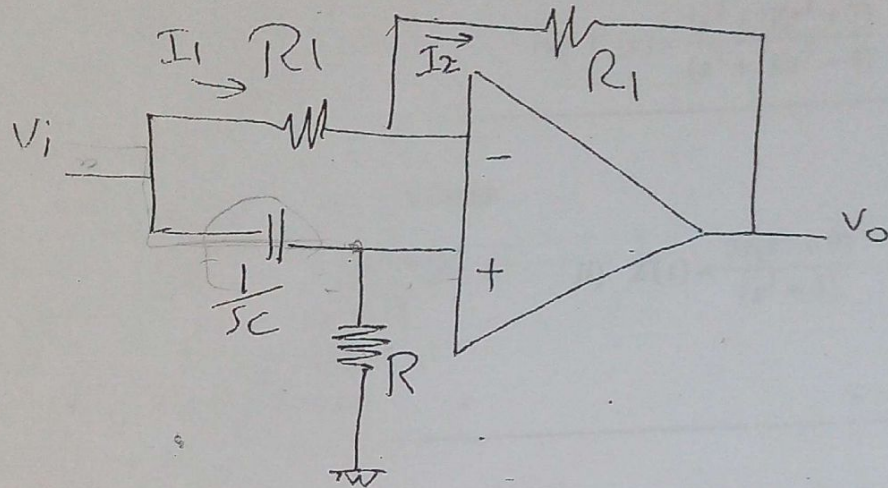
$$R = 10^4 \tan \left(-\frac{\phi}{2} \right)$$

ϕ هو الزاوية

ϕ	R	ϕ	R
-30	2.68 k Ω	-90	10 k Ω
-60	5.77 k Ω	-120	17.32 k Ω
		-150	37.32 k Ω

(14) Show That by interchanging R and C in op-Amp
The resulting phase shift covers The range 0 to 180°
with 0 at high frequency and 180° at low frequency

Solution:-



$$V^+ = V_i \frac{R}{R + \frac{1}{sC}} = V_i \frac{sCR}{sCR + 1}$$

$$I_1 = I_2 \quad V_o = -\frac{R}{R_1} V^+ + V_i \frac{sCR}{sCR + 1}$$

$$\frac{V_i - V^+}{R} = \frac{V^+ - V_o}{R} \quad \frac{V_o}{V_i} = -1 + \frac{RCs}{RCs + 1}$$

$$V_i - V^+ = V^+ - V_o$$

$$V_i - V_i \frac{sCR}{sCR + 1} = V_i \frac{sCR}{sCR + 1} - V_o$$

Sheet #2

Two Terminal LC Network SynthesisQ.1

Determine which of the following functions may be a driving point impedance or admittance function for an LC network:

$$i) Z(s) = \frac{2s(s^2+2)(s^2+5)}{(s^2+1)(s^2+3)}$$

$$ii) Z(s) = \frac{2s^2+2s}{(s^2+4)}$$

$$iii) Y(s) = \frac{s^4+6s^2+5}{s^3+3s}$$

$$iv) Y(s) = \frac{(s^2+1)(s^2+3)}{(s^2+1)(s^2+4)}$$

Q.2

Synthesize the following impedance functions:

$$i) Z(s) = \frac{(s^2+1)(s^2+3)}{s(s^2+2)}$$

$$ii) Z(s) = \frac{5s(s^2+9)}{(s^2+4)}$$

a- Using first Foster method.

b- Using second Cauer method.

Q.3

Synthesize the following driving point functions using First Foster, First and Second Cauer forms:

$$i) Z(s) = \frac{2(s^2+1)(s^2+4)(s^2+6)}{s(s^2+2)(s^2+5)}$$

$$ii) Y(s) = \frac{s(s^2+2)(s^2+4)}{(s^2+1)(s^2+3)}$$

$$iii) Z(s) = \frac{5s(s^2+9)}{s^2+4}$$

$$iv) Y(s) = \frac{3s(s^2+2)(s^2+5)(s^2+9)}{(s^2+1)(s^2+4)(s^2+7)}$$

Q.4

Synthesize the following impedance function in Foster's second form and sketch its pole zero diagram.

$$Z(s) = \frac{(2s^4+8s^2+6)(s^2+5)}{s^5+6s^3+8s}$$

Q.5

Synthesize the following impedance function in Cauer's first form. Illustrate the process of pole removal with the aid of pole zero diagram.

$$Z(s) = \frac{(s^2+1)(s^2+3)}{s(s^2+2)(s^2+4)}$$

Sheet "2"

LC network synthesis

1

$$(9) Z(s) = \frac{2s(s^2+2)(s^2+5)}{(s^2+1)(s^2+3)}$$

الجزء

LC network synthesis

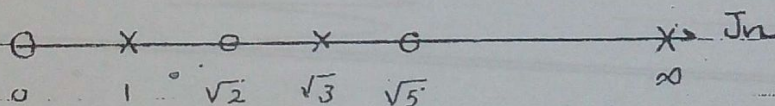
$$(1) Z(s) = \frac{2s(s^4+7s^2+10)}{s^4+4s^2+3} = \frac{2s^5+14s^3+20s}{s^4+4s^2+3} = \frac{\text{odd}}{\text{even}}$$

(2) The Coefficients (الاعمال) are positive real
 (3/4/1) (2/14/2) (3/4/1) (2/14/2)

(3) The poles and zeros are simple and interlacing

$$(4) \text{ Poles: } s = \pm j, s = \pm j\sqrt{3}, s = \infty$$

$$\text{Zeros: } s = 0, s = \pm j\sqrt{2}, s = \pm j\sqrt{5}$$



The Poles and Zeros are simple, in Conjugate Pairs and lie on the jw axis.

The Poles and Zeros are interlacing

$$(5) Z(jw) = \frac{-2jw^5 - 14jw^3 + 20jw}{w^4 + 4w^2 + 3} = j \frac{-2w^5 - 14w^3 + 20w}{w^4 + 4w^2 + 3}$$

$$Z(j\omega) = jX(\omega) \quad \text{pure imaginary}$$

$$X(-\omega) = j \frac{2\omega^5 + 14\omega^3 - 20\omega}{\omega^4 - 4\omega^2 + 3}$$

$$X(-\omega) = -j \frac{-2\omega^5 - 14\omega^3 + 20\omega}{\omega^4 - 4\omega^2 + 3} = -jX(\omega)$$

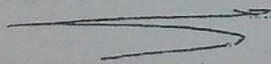
So $X(\omega)$ is odd function of ω

⑥ النقطة هي كذا تربيعية (s^2)

⑦ there must be either pole or zero at the origin and infinity

والنقطة هي كذا تربيعية

So this function is a driving point impedance for LC network.



$$(i) \quad Z(s) = \frac{2s^2 + 2s}{(s^2 + 4)}$$

درجه ی نامبر از درجه ی مخرج

it can't be realized as LC network.

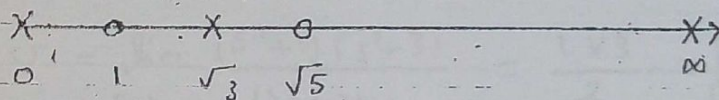
$$(ii) \quad Y(s) = \frac{s^4 + 6s^2 + 5}{s^3 + 3s} = \frac{(s^2 + 1)(s^2 + 5)}{s(s^2 + 3)}$$

$$(i) \quad Y(s) = \frac{\text{even}}{\text{odd}}$$

7

درجه ی نامبر از درجه ی مخرج

$$(2) \quad \begin{array}{l} \text{Poles: } s = 0, s = \pm j\sqrt{3}, s = \infty \\ \text{Zeros: } s = \pm j, s = \pm j\sqrt{5} \end{array}$$



the Poles and Zeros are simple, and interlacing and lie on the $j\omega$ axis

(3) there is pole at $s=0$ and pole at $s=\infty$

(4) the coefficients are positive real

(5) reciprocal

so it is a driving point admittance for LC network

$$(iv) \quad Y(s) = \frac{(s^2 + 1)(s^2 + 3)}{(s^2 + 1)(s^2 + 4)}$$

درجه ی نامبر از درجه ی مخرج

so it is not driving point for LC network.

2. Synthesize the following impedance functions

(a) using Foster I

(b) using Cauer II

$$(c) Z(s) = \frac{(s^2+1)(s^2+3)}{s(s^2+2)}$$

Foster I

$$Z(s) = Hs + \frac{K_0}{s} + \frac{K_1 s}{s^2+2}$$

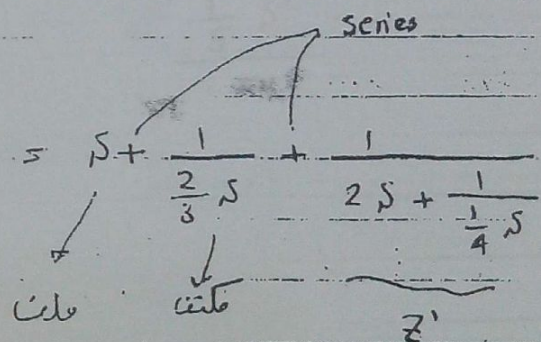
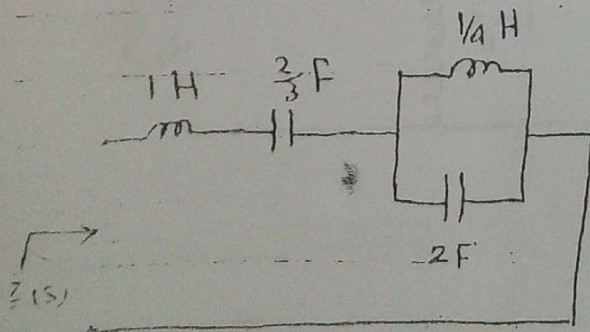
$$H = \lim_{s \rightarrow \infty} \frac{Z(s)}{s} = \lim_{s \rightarrow \infty} \frac{(s^2+1)(s^2+3)}{s^2(s^2+2)} = \lim_{s \rightarrow \infty} \frac{(1+1/s^2)(1+3/s^2)}{1(1+2/s^2)} = \boxed{1}$$

$$K_0 = \lim_{s \rightarrow 0} s Z(s) = \lim_{s \rightarrow 0} \frac{(s^2+1)(s^2+3)}{(s^2+2)} = \frac{1 \times 3}{2} = \boxed{\frac{3}{2}}$$

$$K_1 = \lim_{s^2 \rightarrow -2} (s^2+2) \frac{Z(s)}{s} = \lim_{s^2 \rightarrow -2} \frac{(s^2+1)(s^2+3)}{s^2} = \frac{(-1)(1)}{-2} = \boxed{\frac{1}{2}}$$

$$\therefore Z(s) = s + \frac{3/2}{s} + \frac{1/2 s}{s^2+2}$$

$$= s + \frac{1}{\frac{2}{3}s} + \frac{1}{2s + \frac{4}{s}}$$



$$Y' = \frac{1}{Z'} = 2s + \frac{1}{\frac{1}{4}s}$$

parallel

Cauer II

it is as removing pole at the origin
 so it must be s in the numerator

چون که می‌خواهیم پل در مبدأ را حذف کنیم
 پس باید s در صورت کسر باشد

$$Z(s) = \frac{(s^2+1)(s^2+3)}{s(s^2+2)} = \frac{s^4 + 4s^2 + 3}{s^3 + 2s} = \frac{3 + 4s^2 + s^4}{2s + s^3}$$

$$\begin{array}{r} 2s + s^3 \overline{) 3 + 4s^2 + s^4} \\ \underline{3 + \frac{3}{2}s^2} \\ \frac{5}{2}s^2 + s^4 \end{array}$$

Z_1

مکث

$$\begin{array}{r} \frac{5}{2}s^2 + s^4 \overline{) 2s + s^3} \\ \underline{2s + \frac{4}{5}s^3} \\ \frac{1}{5}s^3 \end{array}$$

Y_2

مکث

$$\begin{array}{r} \frac{1}{5}s^3 \overline{) \frac{5}{2}s^2 + s^4} \\ \underline{\frac{5}{2}s^2 + s^4} \\ \phantom{\frac{5}{2}s^2 + } \frac{25}{2}s \end{array}$$

Z_3

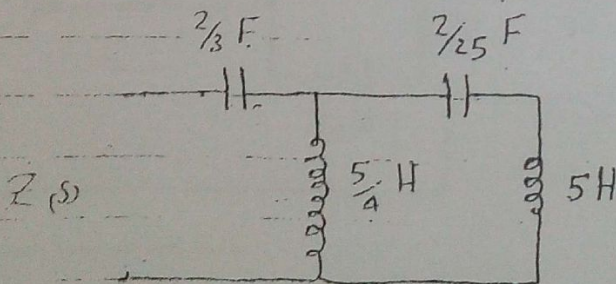
مکث

$$\frac{5}{2}s^2$$

$$\begin{array}{r} s^4 \overline{) \frac{1}{5}s^3} \\ \underline{\frac{1}{5}s^3} \\ \phantom{\frac{1}{5}s^3} \frac{1}{5}s^3 \end{array}$$

Y_4

$$\frac{1}{5}s^3$$



$$(22) \quad Z(s) = \frac{5s(s^2+9)}{(s^2+4)}$$

(Faste I)

$$Z(s) = Hs + \frac{k_0}{s} + \frac{k_1 s}{s^2+4}$$

$$H = \lim_{s \rightarrow \infty} \frac{Z(s)}{s} = \lim_{s \rightarrow \infty} \frac{5(s^2+9)}{(s^2+4)} = \frac{5(1+9/s^2)}{(1+4/s^2)} = \boxed{5}$$

$$k_0 = \lim_{s \rightarrow 0} s Z(s) = \lim_{s \rightarrow 0} \frac{5s^2(s^2+9)}{(s^2+4)} = \boxed{0}$$

$$k_1 = \lim_{s^2=-4} (s^2+4) \frac{Z(s)}{s} = \lim_{s^2=-4} 5(s^2+9) = 5 \times 5 = \boxed{25}$$

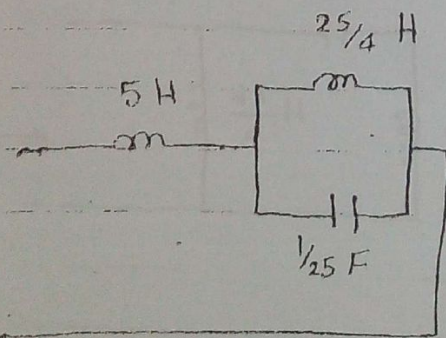
$$Z(s) = 5s + \frac{25s}{s^2+4} = 5s + \frac{1}{\frac{1}{25}s + \frac{4}{25s}}$$

$$= 5s + \frac{1}{\frac{1}{25}s + \frac{1}{\frac{25}{4}s}}$$

Z'

$$Y' = \frac{1}{Z'} = \frac{1}{25}s + \frac{1}{\frac{25}{4}s}$$

ملف ←
ملف ←



Ques II

$$Z(s) = \frac{5s(s^2+9)}{(s^2+4)}$$

حساب آنیم بکسر حذف الی که معروده مندرجانیست
برای آن نقیصه را از آن کم می کنیم

$$Y(s) = \frac{1}{Z(s)} = \frac{s^2+4}{5s(s^2+9)}$$

$$= \frac{s^2+4}{5s^3+45s} = \frac{4+s^2}{45s+5s^3}$$

$$45s+5s^3 \overline{) 4+s^2} \quad \left| \frac{4}{45s} \right.$$

$$4 + \frac{4}{9}s^2$$

Y_1

مقدار

$$\frac{5}{9}s^2 \overline{) 45s+5s^3} \quad \left| \frac{81}{s} \right.$$

$$45s$$

Z_1

مقدار

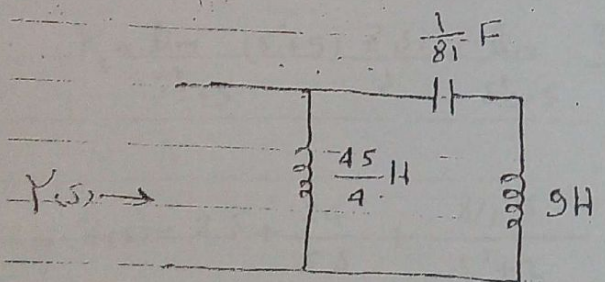
$$5s^3 \overline{) \frac{5}{9}s^2} \quad \left| \frac{1}{9s} \right.$$

$$\frac{5}{9}s^2$$

$$0$$

Y_3

مقدار



(8)

[5] use Foster I, Cauer I, Cauer II

$$(i) Z(s) = \frac{2(s^2+1)(s^2+4)(s^2+6)}{s(s^2+2)(s^2+5)}$$

Foster I

$$Z(s) = Hs + \frac{k_0}{s} + \frac{k_1 s}{(s^2+2)} + \frac{k_2 s}{(s^2+5)}$$

$$H = \lim_{s \rightarrow \infty} \frac{Z(s)}{s} = \lim_{s \rightarrow \infty} \frac{2(s^2+1)(s^2+4)(s^2+6)}{s^2(s^2+2)(s^2+5)} = \boxed{2}$$

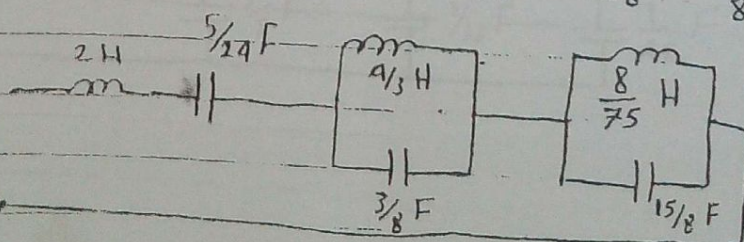
$$k_0 = \lim_{s \rightarrow 0} s Z(s) = \lim_{s \rightarrow 0} \frac{2(s^2+1)(s^2+4)(s^2+6)}{(s^2+2)(s^2+5)} = \frac{2 \times 4 \times 6}{2 \times 5} = \boxed{\frac{24}{5}}$$

$$k_1 = \lim_{s^2 \rightarrow -2} (s^2+2) Z(s) = \lim_{s^2 \rightarrow -2} \frac{2(s^2+1)(s^2+4)(s^2+6)}{s^2(s^2+5)} = \frac{2(-1)(2)(4)}{(-2)(3)} = \boxed{\frac{8}{3}}$$

$$k_2 = \lim_{s^2 \rightarrow -5} (s^2+5) Z(s) = \lim_{s^2 \rightarrow -5} \frac{2(s^2+1)(s^2+4)(s^2+6)}{s^2(s^2+2)} = \frac{2(-4)(-1)(1)}{-5(-3)} = \boxed{1}$$

$$Z(s) = 2s + \frac{24}{5s} + \frac{8/3 s}{s^2+2} + \frac{8/15 s}{s^2+5}$$

$$= 2s + \frac{1}{\frac{5}{24}s} + \frac{1}{\frac{3}{8}s + \frac{3}{4s}} + \frac{1}{\frac{15}{8}s + \frac{75}{8s}}$$



(Case I) $Z(s) = \frac{2(s^2+1)(s^2+4)(s^2+6)}{s(s^2+2)(s^2+5)}$

$$2s^6 + 22s^4 + 68s^2 + 48$$

$$s^5 + 7s^3 + 10s$$

نصف البسط اعلى من المقام

$$\begin{array}{r} s^5 + 7s^3 + 10s \overline{) 2s^6 + 22s^4 + 68s^2 + 48} \\ \underline{2s^6 + 14s^4 + 20s^2} \\ 8s^4 + 48s^2 + 48 \end{array} \quad \begin{array}{l} 2s \\ Z_1 \end{array}$$

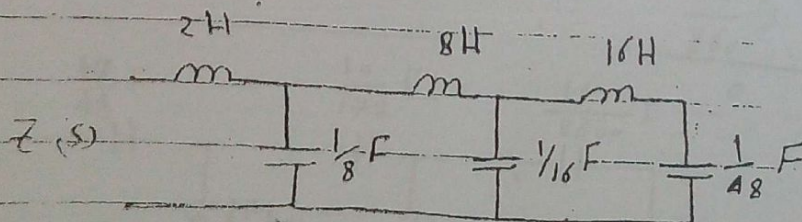
$$\begin{array}{r} 8s^4 + 48s^2 + 48 \overline{) s^5 + 7s^3 + 10s} \\ \underline{s^5 + 6s^3 + 6s} \\ 8s^4 + 48s^2 + 48 \end{array} \quad \begin{array}{l} \frac{1}{8}s \\ Y_2 \end{array}$$

$$\begin{array}{r} s^3 + 4s \overline{) 8s^4 + 48s^2 + 48} \\ \underline{8s^4 + 32s^2} \\ 16s^2 + 48 \end{array} \quad \begin{array}{l} 8s \\ Z_3 \end{array}$$

$$\begin{array}{r} 16s^2 + 48 \overline{) s^3 + 4s} \\ \underline{s^3 + 3s} \\ 16s^2 + 48 \end{array} \quad \begin{array}{l} \frac{1}{16}s \\ Y_4 \end{array}$$

$$\begin{array}{r} s \overline{) 16s^2 + 48} \\ \underline{16s^2} \\ 48 \end{array} \quad \begin{array}{l} 16s \\ Z_5 \end{array}$$

$$\begin{array}{r} 48 \overline{) s} \\ \underline{s} \\ 0 \end{array} \quad \begin{array}{l} \frac{1}{48}s \\ Y_6 \end{array}$$



Exercice II

سبب الترددات المعقدة من الختام

$$Z(s) = \frac{48 + 68s^2 + 22s^4 + 2s^6}{10s + 7s^3 + s^5}$$

$$10s + 7s^3 + s^5 \overline{) 48 + 68s^2 + 22s^4 + 2s^6} \quad \begin{array}{r} 48 \\ 10s \end{array}$$

$$48 + \frac{168}{5}s^2 + \frac{24}{5}s^4$$

Z_1

مكت

$$\frac{172}{5}s^2 + \frac{86}{5}s^4 + 2s^6$$

$$10s + 7s^3 + s^5 \overline{) \frac{50}{172s}}$$

$$10s + \frac{860}{172}s^3 + \frac{100s^5}{172}$$

Y_2

مكت

$$2s^3 + \frac{72}{172}s^5 \overline{) \frac{172}{5}s^2 + \frac{86}{5}s^4 + 2s^6} \quad \begin{array}{r} 172 \\ 10s \end{array}$$

$$\frac{172}{5}s^2 + \frac{72}{10}s^4$$

$$10s^4 + 2s^6$$

$$10s^4 + 2s^6 \overline{) 2s^3 + \frac{72}{172}s^5} \quad \begin{array}{r} 1 \\ 5s \end{array}$$

$$2s^3 + \frac{2}{5}s^5$$

Y_4

مكت

$$\frac{16}{860}s^5 \overline{) 10s^4 + 2s^6} \quad \begin{array}{r} 8600 \\ 16s \end{array}$$

$$10s^4 +$$

Z_5

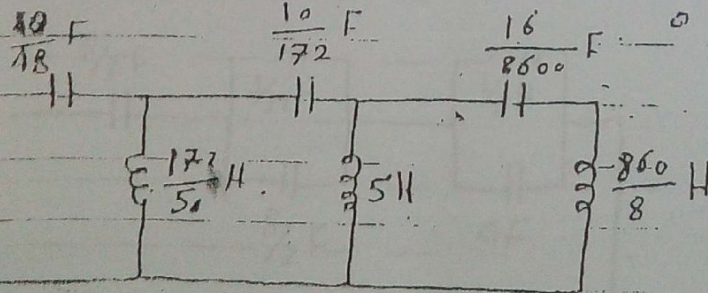
مكت

$$2s^6 \overline{) \frac{16}{860}s^5} \quad \begin{array}{r} 8 \\ 860s \end{array}$$

Y_6

مكت

$$\frac{16}{860}s^5$$



(ii) $Y(s) = \frac{s(s^2+2)(s^2+4)}{(s^2+1)(s^2+3)}$

(Foster I) it is used to synthesize impedance function $Z(s)$

Let $Z(s) = \frac{1}{Y(s)} = \frac{(s^2+1)(s^2+3)}{s(s^2+2)(s^2+4)}$

$Z(s) = H + \frac{K_0}{s} + \frac{K_1 s}{s^2+2} + \frac{K_2 s}{s^2+4}$

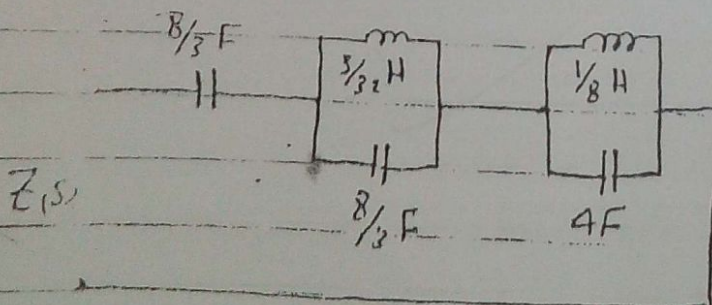
$H = \lim_{s \rightarrow \infty} \frac{Z(s)}{s} = \lim_{s \rightarrow \infty} \frac{(s^2+1)(s^2+3)}{s^2(s^2+2)(s^2+4)} = \frac{\frac{1}{s^2} (1+\frac{1}{s^2})(1+\frac{3}{s^2})}{1(1+\frac{2}{s^2})(1+\frac{4}{s^2})} = \boxed{0}$

$K_0 = \lim_{s \rightarrow 0} s Z(s) = \lim_{s \rightarrow 0} \frac{(s^2+1)(s^2+3)}{(s^2+2)(s^2+4)} = \frac{1 \times 3}{2 \times 4} = \boxed{\frac{3}{8}}$

$K_1 = \lim_{s^2 \rightarrow -2} \frac{Z(s)}{(s^2+2)} = \lim_{s^2 \rightarrow -2} \frac{(s^2+1)(s^2+3)}{s^2(s^2+4)} = \frac{(-1)(1)}{-2(2)} = \boxed{\frac{1}{4}}$

$K_2 = \lim_{s^2 \rightarrow -4} \frac{Z(s)}{(s^2+4)} = \lim_{s^2 \rightarrow -4} \frac{(s^2+1)(s^2+3)}{s^2(s^2+2)} = \frac{(-3)(-1)}{-4(-2)} = \boxed{\frac{3}{8}}$

$Z(s) = \frac{3/8}{s} + \frac{1/4 s}{s^2+2} + \frac{3/8 s}{s^2+4} = \frac{3/8}{s} + \frac{1}{\frac{8}{3}s + \frac{32}{3}} + \frac{1}{\frac{4s+8}{s}}$



Case I $Y(s) = \frac{s(s^2+2)(s^2+4)}{(s^2+1)(s^2+3)} = \frac{s(s^4+6s^2+8)}{s^4+4s^2+3}$

$$Y(s) = \frac{s^5+6s^3+8s}{s^4+4s^2+3}$$

درجه الب > اقل من درجه المقام
لذلك نكتبها على شكل Y

$$s^4+4s^2+3 \overline{) s^5+6s^3+8s} \quad Y_1$$

مكتمل

$$s+4s^3+3s$$

$$2s^3+5s \overline{) s^4+4s^2+3} \quad Y_2$$

مكتمل

$$s^4+\frac{5}{2}s^2$$

$$\frac{3}{2}s^2+3 \overline{) 2s^3+5s} \quad Y_3$$

مكتمل

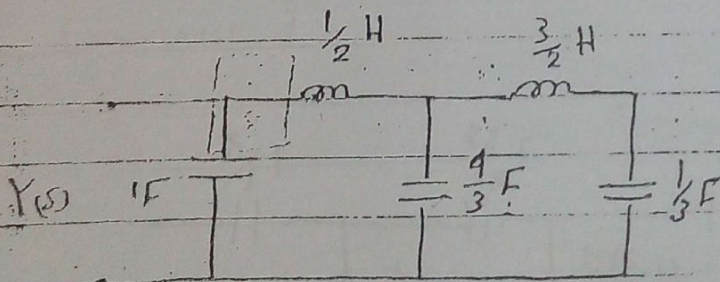
$$2s^3+4s$$

$$s \overline{) \frac{3}{2}s^2+3} \quad Y_4$$

$$\frac{3}{2}s^2$$

$$3 \overline{) s} \quad \frac{1}{3}s$$

$$s$$



(Circuit II) $Y(s) = \frac{s(s^2+2)(s^2+4)}{(s^2+1)(s^2+3)}$

$$Z(s) = \frac{(s^2+1)(s^2+3)}{s(s^2+2)(s^2+4)}$$

$$Z(s) = \frac{3s^4 + 4s^2 + 34}{8s^5 + 6s^3 + s}$$

أولاً توجد دالة متعادلة من المقام

في المقام $Z(s)$

بوجود Pole عند $s=0$ و removed.

ثم ترتيبها ترتيباً تصاعدياً

~~$$3 + 4s^2 + s^4 \mid 8s^5 + 6s^3 + s \mid \frac{8}{3s}$$~~

Z_1

$$8s^5 + 6s^3 + s \mid 3 + 4s^2 + s^4 \mid \frac{3}{8s}$$

$$\frac{1}{8} + \frac{9}{4}s^2 + \frac{3}{8}s^4$$

Z_1 مكتمل

$$\frac{7}{4}s^2 + \frac{5}{8}s^4 \mid 8s^5 + 6s^3 + s \mid \frac{32}{7s}$$

$$8s^5 + 4s^3$$

Z_2

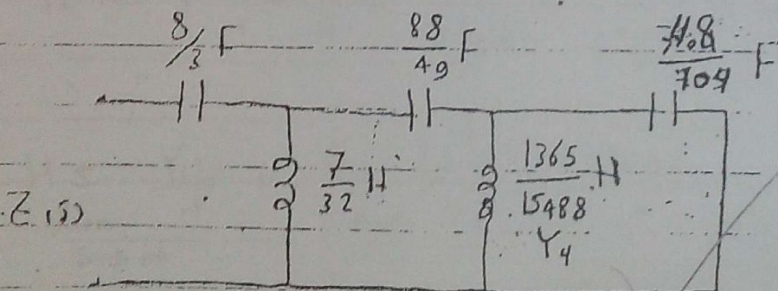
$$\frac{22}{7}s^3 + s^5 \mid \frac{7}{4}s^2 + \frac{5}{8}s^4 \mid \frac{49}{88s}$$

$$\frac{7}{4}s^3 + \frac{49}{88}s^4$$

Z_3

$$48s^4 \mid 22s^3 + s^5 \mid \frac{(704) \times 21}{s(7848)}$$

$$704 \mid 22s^3 + s^5 \mid 1335$$



$$s^5 \mid 48s^4 \mid \frac{48}{704}$$

$$48s^4 \mid 704 \mid 0$$

$$[1] \quad Z(s) = \frac{(2s^4 + 8s^2 + 6)(s^2 + 5)}{s^5 + 6s^3 + 8s}$$

using Foster II

$$Z(s) = \frac{2(s^4 + 4s^2 + 3)(s^2 + 5)}{s(s^4 + 6s^2 + 8)} = \frac{2(s^2 + 1)(s^2 + 3)(s^2 + 5)}{s(s^2 + 2)(s^2 + 4)}$$

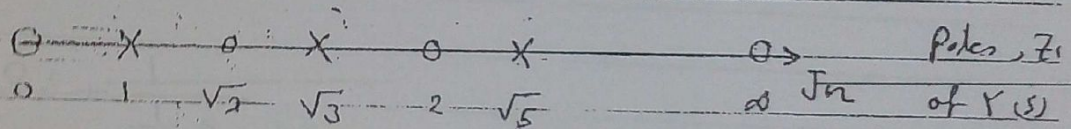
For Foster II it must be admittance $Y(s)$

$$Y(s) = \frac{s(s^2 + 2)(s^2 + 4)}{2(s^2 + 1)(s^2 + 3)(s^2 + 5)}$$

لأمر درج اولی
این صفر درجه اولی

Zeros: $s = 0, s = \pm j\sqrt{2}, s = \pm j2, s = \infty$

Poles: $s = \pm j, s = \pm j\sqrt{3}, s = \pm j\sqrt{5}$



(pole-zero diagram)

$$Y(s) = H \left[\frac{k_0}{s} + \frac{k_1 s}{s^2 + 1} + \frac{k_2 s}{s^2 + 3} + \frac{k_3 s}{s^2 + 5} \right]$$

$$H = \lim_{s \rightarrow \infty} \frac{Y(s)}{s} = \lim_{s \rightarrow \infty} \frac{(s^2 + 2)(s^2 + 4)}{2(s^2 + 1)(s^2 + 3)(s^2 + 5)} = \boxed{0}$$

$$k_0 = \lim_{s \rightarrow 0} s Y(s) = \lim_{s \rightarrow 0} \frac{s^2 (s^2 + 2)(s^2 + 4)}{2(s^2 + 1)(s^2 + 3)(s^2 + 5)} = \boxed{0}$$

$$k_1 = \lim_{s^2 \rightarrow -1} (s^2 + 1) \frac{Y(s)}{s} = \lim_{s^2 \rightarrow -1} \frac{(s^2 + 2)(s^2 + 4)}{2(s^2 + 3)(s^2 + 5)} = \frac{(1)(3)}{2(2)(4)} = \boxed{\frac{3}{16}}$$

$$K_2 = \lim_{s^2=-3} (s^2+3) \frac{Y(s)}{s} = \lim_{s^2=-3} \frac{(s^2+2)(s^2+4)}{2(s^2+1)(s^2+5)} = \frac{(-1)(1)}{2(-2)(2)}$$

$$K_3 = \lim_{s^2=-5} (s^2+5) \frac{Y(s)}{s} = \lim_{s^2=-5} \frac{(s^2+2)(s^2+4)}{2(s^2+1)(s^2+3)} = \frac{(-3)(-1)}{2(-4)(-2)}$$

$$Y(s) = \frac{3/16 s}{s^2+1} + \frac{1/8 s}{s^2+3} + \frac{3/16 s}{s^2+5}$$

$$= \underbrace{\frac{1}{\frac{16}{3}s + \frac{16}{3s}}}_{Y_1} + \underbrace{\frac{1}{8s + \frac{24}{s}}}_{Y_2} + \underbrace{\frac{1}{\frac{16}{3}s + \frac{80}{3s}}}_{Y_3}$$

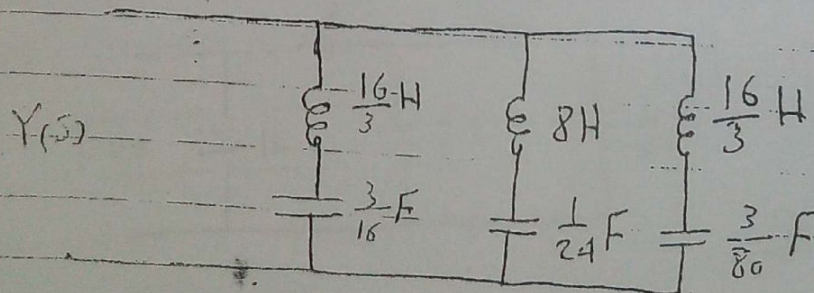
Parallel

$$Z_1 = \frac{16}{3}s + \frac{1}{\frac{3}{16}s} \rightarrow \omega \omega$$

Series

$$Z_2 = 8s + \frac{1}{\frac{1}{24}s}$$

$$Z_3 = \frac{16}{3}s + \frac{1}{\frac{3}{80}s}$$



using Cramer's

$$Z(s) = \frac{(s^2+1)(s^2+3)}{s(s^2+2)(s^2+4)}$$

$$Z(s) = \frac{s^4 + 4s^2 + 3}{s(s^4 + 6s^2 + 8)} = \frac{s^4 + 4s^2 + 3}{s^5 + 6s^3 + 8s}$$

لذلك يجب علينا ان نكتب $Y(s)$ على شكل

$$Y(s) = \frac{s^5 + 6s^3 + 8s}{s^4 + 4s^2 + 3} = \frac{s(s^2+2)(s^2+4)}{(s^2+1)(s^2+3)}$$

$$\begin{array}{r} s^4 + 4s^2 + 3 \overline{) s^5 + 6s^3 + 8s} \quad s \\ \hline 2s^3 + 5s \end{array}$$

Y_1 مكتمل

$$\begin{array}{r} 2s^3 + 5s \overline{) s^4 + 4s^2 + 3} \quad \frac{1}{2}s \\ \hline s^4 + \frac{5}{2}s^2 \end{array}$$

Z_2 ملف

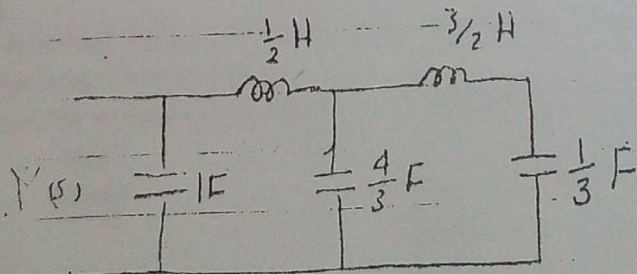
$$\begin{array}{r} \frac{3}{2}s^2 + 3 \overline{) 2s^3 + 5s} \quad \frac{4}{3}s \\ \hline 2s^3 + 4s \end{array}$$

Y_3

$$\begin{array}{r} s \overline{) \frac{3}{2}s^2 + 3} \quad \frac{3}{2}s \\ \hline \frac{3}{2}s^2 \end{array}$$

Z

$$\begin{array}{r} 3 \overline{) s} \quad \frac{1}{3}s \\ \hline s \end{array}$$



OP

Sheet #3

Two Terminal RC Network Synthesis

RC

Q.1

Synthesize the first and second forms of the Foster networks for the following impedance functions: -

$$i) Z(s) = \frac{(s+2)(s+5)}{(s+1)(s+3)} \quad / \quad ii) Z(s) = \frac{(s+2.5)(s+10)}{(s+1)(s+7.5)}$$

Q.2

Synthesize the first and second Cauer forms for the networks having the following impedance functions.

$$a) Y(s) = \frac{2s + s^2}{3 + 4s + s^2}$$

$$b) Z(s) = \frac{s^2 + 7s + 4}{3s^2 + 2s}$$

Q.3

Synthesize the following impedance functions using the four canonical forms.

$$a) Y(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)}$$

$$b) Z(s) = 2 \frac{(s+2)(s+4)(s+6)}{s(s+3)(s+5)}$$

$$c) Z(s) = \frac{8(s+1)(s+2)(s+3)}{s(2s+3)(2s+5)(2s+7)}$$

Cauer 1



Q.4

An impedance function has a simple poles at -1 and -4 and simple zeroes at -2 and -5. $Z(0) = 10 \Omega$. Find the four canonical networks for this $Z(s)$ with element values.

Q.5

A four element ladder network has a minimum number of elements to realize a given $Z(s)$. The two resistors have the value of 4 ohms and the two capacitors 4 farads. What are the possible forms of the driving point impedances $Z(s)$?

Q.6

a) Show that $Z_{RL}(s)$, the driving point impedance of the general RL one port network, has the same form as $Y_{RC}(s)$.

b) Use the results of part a to find the four networks of the function $F(s)$ interpreted as

i) $Z(s)$ ii) $Y(s)$

where:

$$F(s) = \frac{(s+1)(s+3)}{s(s+2)}$$

III sheet "3"

1) Synthesize Foster I and Foster II networks

$$(a) \quad Z(s) = \frac{(s+2)(s+5)}{(s+1)(s+3)}$$

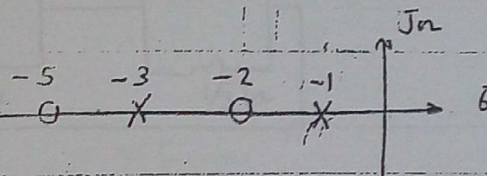
أولاً نتحقق

$$* \quad Z(s) = \frac{P_n(s)}{Q_n(s)}$$

درجته البسط مساوية لدرجة المقام

$$* \quad \text{Poles} \quad s = -1 \quad \text{و} \quad s = -3$$

$$\text{Zeros} \quad s = -2 \quad \text{و} \quad s = -5$$



Poles and Zeros are -ve real, interlacing

* start with pole

ended with Zero

so it is RC network

Foster I

$$Z(s) = K_\infty + \frac{K_0}{s} + \frac{K_1}{s+1} + \frac{K_2}{s+3}$$

$$K_\infty = \lim_{s \rightarrow \infty} Z(s) = \lim_{s \rightarrow \infty} \frac{(s+2)(s+5)}{(s+1)(s+3)} = \boxed{1}$$

$$K_0 = \lim_{s \rightarrow 0} s Z(s) = \lim_{s \rightarrow 0} \frac{s(s+2)(s+5)}{(s+1)(s+3)} = \boxed{0}$$

$$K_1 = \lim_{s \rightarrow -1} (s+1) Z(s) = \lim_{s \rightarrow -1} \frac{(s+2)(s+5)}{(s+3)} = \frac{1 \times 4}{2} = \boxed{2}$$

(2)

$$\lim_{s \rightarrow -2} (s+2) Z(s) = \lim_{s \rightarrow -2} \frac{(s+2)(s+5)}{(s+1)} \cdot \frac{(-1)(1)}{-2} = \boxed{1}$$

$$Z(s) = 1 + \frac{2}{s+1} + \frac{1}{s+3} = 1 + \frac{1}{\frac{s}{2} + \frac{1}{2}} + \frac{1}{s+3}$$

$\underbrace{\hspace{1cm}}_{Z'} \quad \underbrace{\hspace{1cm}}_{Z''}$

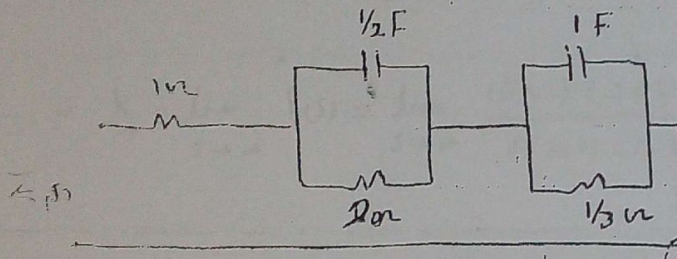
$$Y' = \frac{1}{Z'} = \frac{1}{\frac{s}{2} + \frac{1}{2}}$$

Parallel

 capacitor
 resistor

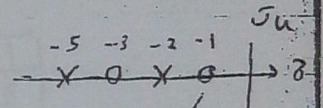
$$Y'' = s+3$$

resistor
 $\frac{1}{R} = 3$
 $R = \frac{1}{3} \Omega$



Foster II

taking $Y(s) = \frac{1}{Z(s)} = \frac{(s+1)(s+3)}{(s+2)(s+5)}$



زیرات
 pole + capacitor
 لا یکنه متبعا

$$Y(s) = K_{\infty} + \frac{K_0}{s} + \frac{K_1}{s+2} + \frac{K_2}{s+5}$$

$$K_{\infty} = \lim_{s \rightarrow \infty} Y(s) = \lim_{s \rightarrow \infty} \frac{(s+1)(s+3)}{(s+2)(s+5)} = \boxed{1}$$

$$K_0 = \lim_{s \rightarrow 0} s Y(s) = \lim_{s \rightarrow 0} \frac{s(s+1)(s+3)}{(s+2)(s+5)} = \boxed{0}$$

$$K_1 = \lim_{s \rightarrow -2} (s+2) Y(s) = \lim_{s \rightarrow -2} \frac{(s+1)(s+3)}{(s+5)} = \frac{(-1)(1)}{3} = \boxed{-\frac{1}{3}}$$

مقاومت سلفی

$$K_2 = \lim_{s \rightarrow -5} (s+5) Y(s) = \lim_{s \rightarrow -5} \frac{(s+1)(s+3)}{(s+2)} = \frac{(-4)(-2)}{-3} = \boxed{-\frac{8}{3}}$$

مقاومت سلفی

لا یکنه متبعا لهذا لا الهات متبعا سلفی، لکن هات متبعا

3

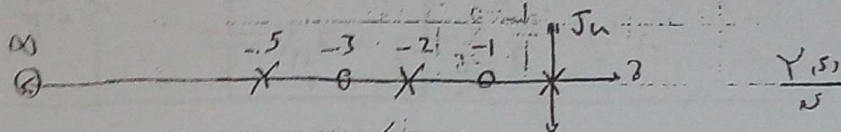
$$\text{Let } P(s) = \frac{Y(s)}{s} = \frac{(s+1)(s+3)}{s(s+2)(s+5)}$$

عند التردد على s

① أوجد مكان pole في $s=0$ (أي بدأت ب pole)

② درجة مقام أعلى من درجة البسط وبالتالي يوجد zero في $s=\infty$

أي (نقطة ب zero)



كلية كوفيتا ك RC

$$K_0 = \lim_{s \rightarrow \infty} P(s) = \lim_{s \rightarrow \infty} \frac{(s+1)(s+3)}{s(s+2)(s+5)} = \boxed{0}$$

$$K_1 = \lim_{s \rightarrow 0} s F(s) = \lim_{s \rightarrow 0} \frac{(s+1)(s+3)}{(s+2)(s+5)} = \frac{1 \times 3}{2 \times 5} = \boxed{\frac{3}{10}}$$

$$K_2 = \lim_{s \rightarrow -2} (s+2) F(s) = \lim_{s \rightarrow -2} \frac{(s+1)(s+3)}{s(s+5)} = \frac{(-1)(1)}{(-2)(3)} = \boxed{\frac{1}{6}}$$

$$K_3 = \lim_{s \rightarrow -5} (s+5) F(s) = \lim_{s \rightarrow -5} \frac{(s+1)(s+3)}{s(s+2)} = \frac{(-4)(-2)}{(-5)(-3)} = \boxed{\frac{8}{15}}$$

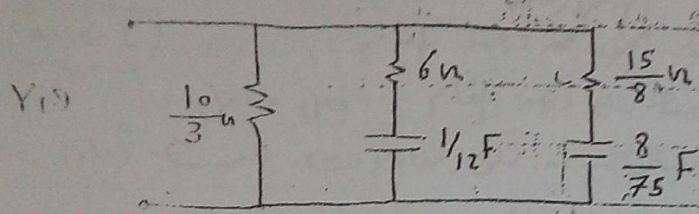
$$\therefore F(s) = \frac{Y(s)}{s} = \frac{3}{10s} + \frac{1/6}{s+2} + \frac{8/15}{s+5}$$

$$Y(s) = \frac{3}{10} + \frac{1/6 s}{s+2} + \frac{8/15 s}{s+5}$$

$$= \frac{3}{10} + \underbrace{\frac{1}{6 + \frac{12}{s}}}_{Y_I} + \underbrace{\frac{1}{\frac{15}{8} + \frac{75}{8s}}}_{Y_{II}}$$

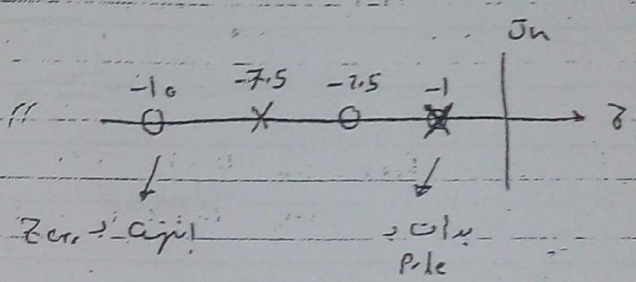
$Z = \frac{1}{\frac{1}{6} + \frac{12}{s}}$
 write
 into

$Z = \frac{15s + 75}{s + 18}$
 (num) 18 : 18s



(11) $Z(s) = \frac{(s+2.5)(s+10)}{(s+1)(s+7.5)}$

Poles $s = -1$ & $s = -7.5$
 Zeros $s = -2.5$ & $s = -10$



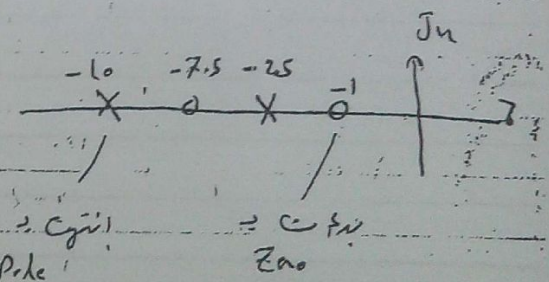
are interlacing

صفر قطب متناهي
 كما في المثال

Foster I

For (Foster II) $Y(s) = \frac{(s+1)(s+7.5)}{(s+2.5)(s+10)}$

Poles $s = -2.5$ & $s = -10$
 Zeros $s = -1$ & $s = -7.5$



لا يمكن تنفيذها مباشرة ولذلك يجب القسمة على s

$F(s) = \frac{Y(s)}{s} \rightarrow$ ثم اعمل أيضا
 كما في المثال

[2]

Case I

Case II

$$(a) Y(s) = \frac{2s + s^2}{3 + 4s + s^2}$$

معادلات المقام بعد
من معادلات البسط
التي هي

Case I

شرطاً ① لا بد أن تكون درجة البسط أقل من درجة المقام

② إذا كانت درجة البسط = درجة المقام نحل معامل

(s^n) آتية أكثر = 1 ثم نضرب أن تكون معادلات البسط أعلى
من نظائرها من معادلات المقام

الشرط الثاني كي يتحقق يجب أن نكتب $Y(s)$ في $Z(s)$

$$Z(s) = \frac{1}{Y(s)} = \frac{s^2 + 4s + 3}{s^2 + 2s}$$

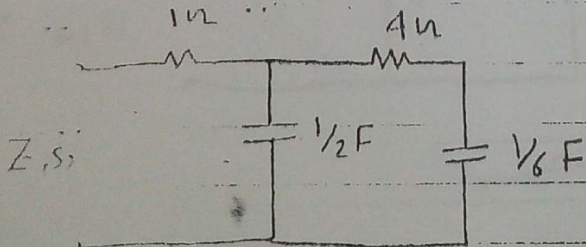
$$\begin{array}{r|l} s^2 + 2s & s^2 + 4s + 3 \quad 1 \quad Z_1 \end{array} \quad \text{مقارنة}$$

$$\begin{array}{r|l} 2s + 3 & s^2 + 2s \quad \frac{1}{2}s \quad Y_2 \end{array} \quad \text{ممكن}$$

$$\begin{array}{r|l} \frac{1}{2}s & 2s + 3 \quad 4 \quad Z_3 \end{array} \quad \text{مقارنة}$$

$$\begin{array}{r|l} 3 & \frac{1}{2}s \quad \frac{1}{6}s \quad Y_4 \end{array} \quad \text{ممكن}$$

$$\frac{\frac{1}{2}s}{s}$$



(6) لا تحقق الشرط

Cauchy II) $Y(s) = \frac{2s + s^2}{3 + 4s + s^2} = \frac{s(2 + s)}{3 + 4s + s^2}$

- ① يجب أن تكون هناك (معادلة) في المقام (Pole) تبدأ بـ (Pole)
 ② إذا لم توجد (معادلة) في المقام ترتيباً ترتيباً لهما
 لنجعل أول معامل = 1 ثم يكون معاملات البسط هي معاملات المقام
 من تبدأ داخلها بـ pole

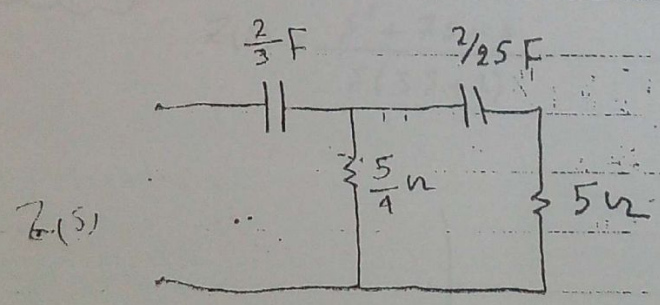
تحقق الشرط $Z(s) = \frac{3 + 4s + s^2}{s(s+2)} = \frac{3 + 4s + s^2}{2s + s^2}$

مكتبة $\frac{2s + s^2}{3 + 4s + s^2} \left| \frac{3}{-2s} \right. Z_1$
 $3 + \frac{3}{2}s$

مقارنة $\rightarrow \frac{5}{2}s + s^2 \left| \frac{4}{5} \right. Y_2$
 $2s + \frac{4}{5}s^2$

$\frac{1}{5}s^2 \left| \frac{25}{2s} \right. Z_3$
 $\frac{5}{2}s$

$s^2 \left| \frac{1}{5}s^2 \right| \frac{1}{5} Y_4$
 $\frac{1}{5}s^2$



(2)

K
↓

$$(b) \quad Z(s) = \frac{s^2 + 7s + 4}{3s^2 + 2s} = \frac{1}{3} \left(\frac{s^2 + 7s + 4}{s^2 + \frac{2}{3}s} \right)$$

معادلات البسط
أكبر من معادلات
المقام
وهو تنفذ كما يلي:-

$$\frac{3s^2 + 2s}{s^2 + \frac{2}{3}s} \overline{) s^2 + 7s + 4} \quad \frac{1}{3} \quad Z_1$$

$$\frac{\frac{19}{3}s + 4}{3s^2 + 2s} \overline{) \frac{19}{3}s} \quad Y_2$$

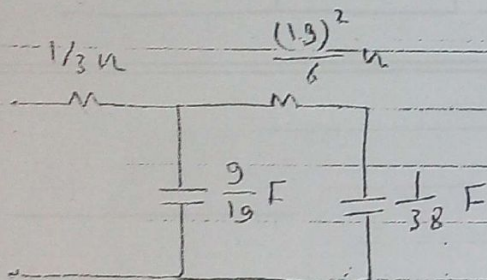
$$3s^2 + 36s$$

$$\frac{\frac{2}{19}s}{\frac{19}{3}s + 4} \overline{) \frac{(19)^2}{6}} \quad Z_3$$

$$\frac{19}{3}s$$

$$\frac{4}{\frac{2}{19}s} \overline{) \frac{1}{38}s} \quad Y_4$$

$$\frac{2}{19}s$$



Case II

$$Z(s) = \frac{s^2 + 7s + 4}{s(3s + 2)}$$

توجد كـ معرودة من المقام

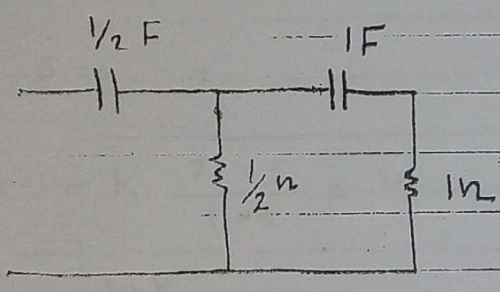


$$Z_1 = \frac{(2s + 3s^2) \left(4 + 2s + \frac{2}{s} \right)}{4 + 6s}$$

$$Y_2 = \frac{(s + s^2) \left(2s + 3s^2 \right) \left(2 \right)}{2s + 2s^2}$$

$$Z_3 = \frac{s^2 \left(s + s^2 \right) \left(\frac{1}{s} \right)}{s}$$

$$Y = \frac{s^2 \left(s^2 \right) \left(1 \right)}{\frac{s^2}{s}}$$



[4]

Poles $s = -1$ & $s = -4$

For impedance function

Zeros $s = -2$ & $s = -5$

$$Z(0) = 10 \Omega$$

$$Z(s) = K \frac{(s+2)(s+5)}{(s+1)(s+4)}$$

Put $s = 0$

$$Z(0) = K \frac{2 \times 5}{1 \times 4} = 10$$

$$\frac{10K}{4} = 10 \Rightarrow K = 4$$

$$Z(s) = \frac{4(s+2)(s+5)}{(s+1)(s+4)} = \frac{4(s^2 + 7s + 10)}{(s^2 + 5s + 4)}$$

طالع آية
آية طالع
المقام

Foster 1

$$Z(s) = K_0 + \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+4}$$

$$K_0 = \lim_{s \rightarrow \infty} Z(s) = \lim_{s \rightarrow \infty} \frac{4(s+2)(s+5)}{(s+1)(s+4)} = 4$$

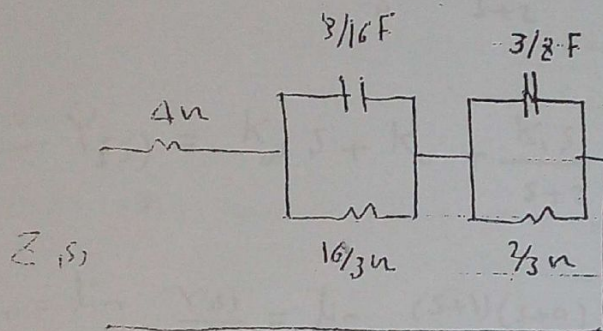
$$K_1 = \lim_{s \rightarrow 0} s Z(s) = \lim_{s \rightarrow 0} \frac{4s(s+2)(s+5)}{(s+1)(s+4)} = 0$$

$$K_2 = \lim_{s \rightarrow -1} (s+1) Z(s) = \lim_{s \rightarrow -1} \frac{4(s+2)(s+5)}{(s+4)} = \frac{4(1)(4)}{3} = \frac{16}{3}$$

$$K_2 = \lim_{s \rightarrow -4} (s+4) \frac{4(s+2)(s+5)}{(s+1)(s+4)} = \frac{4(-2)(1)}{-3} = \frac{8}{3}$$

$$Z(s) = 4 + \frac{16/3}{s+1} + \frac{8/3}{s+4} = 4 + \frac{1}{\frac{3}{16}s + \frac{3}{16}} + \frac{1}{\frac{3}{8}s + \frac{3}{2}}$$

مقاومة
مقاومة
مقاومة



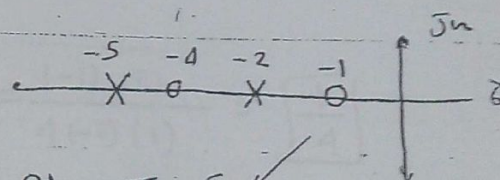
$$Y^1 = \frac{3}{16}s + \frac{3}{16}$$

$$Y^2 = \frac{3}{8}s + \frac{3}{2}$$

Foster II

$$Y(s) = \frac{(s+1)(s+4)}{4(s+2)(s+5)}$$

Poles $s = -2, -5$
Zeros $s = -1, -4$



~~نحوه ساخت مدار فاستر دوم~~

RC شبکه

- ① Poles & Zeros are -ve real
- ② ~ ~ ~ alternating
- ③ start with zero and end with pole

و باید که $k_2 > k_1$ و k_0 را حذف کنیم

For example $F(s) = K_0 + \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+5}$

-ve

$$K_1 = \lim_{s \rightarrow -2} (s+2) Y(s) = \lim_{s \rightarrow -2} \frac{4(s+1)(s+4)}{4(s+5)} = \frac{(-1)(3)}{4(4)} = \frac{-3}{16}$$

So we use $\frac{Y(s)}{s} = \frac{(s+1)(s+4)}{4s(s+2)(s+5)}$

$$\frac{Y(s)}{s} = K_\infty + \frac{K_0}{s} + \frac{K_1}{s+2} + \frac{K_2}{s+5}$$

$$Y(s) = K_\infty s + K_0 + \frac{K_1 s}{s+2} + \frac{K_2 s}{s+5}$$

$$K_\infty = \lim_{s \rightarrow \infty} \frac{Y(s)}{s} = \lim_{s \rightarrow \infty} \frac{(s+1)(s+4)}{4s(s+2)(s+5)} = \boxed{0}$$

$$K_0 = \lim_{s \rightarrow 0} Y(s) = \lim_{s \rightarrow 0} \frac{(s+1)(s+4)}{4(s+2)(s+5)} = \frac{(1)(4)}{4(2)(5)} = \boxed{\frac{1}{10}}$$

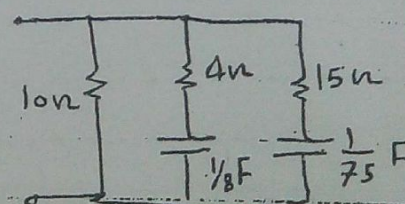
$$K_1 = \lim_{s \rightarrow -2} (s+2) \frac{Y(s)}{s} = \lim_{s \rightarrow -2} \frac{(s+1)(s+4)}{4s(s+5)} = \frac{(-1)(2)}{4(-2)(1)} = \boxed{\frac{1}{4}}$$

$$K_2 = \lim_{s \rightarrow -5} (s+5) \frac{Y(s)}{s} = \lim_{s \rightarrow -5} \frac{(s+1)(s+4)}{4s(s+2)} = \frac{(-4)(-1)}{4(-5)(-3)} = \boxed{\frac{1}{15}}$$

$$Y(s) = \frac{1}{10} + \frac{1/4 s}{s+2} + \frac{1/15 s}{s+5}$$

$$= \frac{1}{10} + \frac{1}{4 + \frac{8}{s}} + \frac{1}{15 + \frac{75}{s}}$$

$\frac{1}{R}$ $Z' = 4 + \frac{8}{s}$ $Z'' = 15 + \frac{75}{s}$



Case I $Z(s) = \frac{4s^2 + 28s + 40}{s^2 + 5s + 4}$ معادلات البسط < معادلات المقام

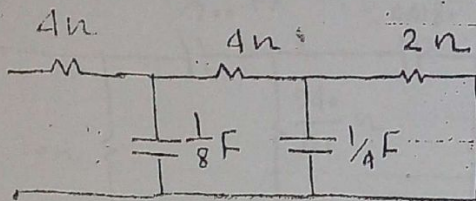
$$\begin{array}{r} s^2 + 5s + 4 \overline{) 4s^2 + 28s + 40} \\ \underline{4s^2 + 20s + 16} \\ 8s + 24 \end{array}$$

$$\begin{array}{r} 8s + 24 \overline{) s^2 + 5s + 4} \\ \underline{s^2 + 8s} \\ -3s + 4 \end{array}$$

$$\begin{array}{r} -3s + 4 \overline{) 8s + 24} \\ \underline{-8s + 16} \\ 40 \end{array}$$

$$\begin{array}{r} 40 \overline{) 8s + 24} \\ \underline{8s} \\ 24 \end{array}$$

$$\begin{array}{r} 24 \overline{) 4} \\ \underline{24} \\ 0 \end{array}$$



Case II

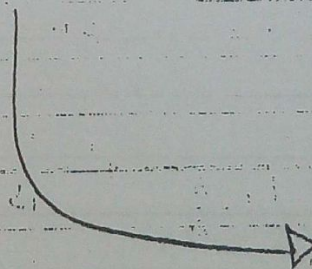
$$Z(s) = \frac{1 + \frac{28}{40}s + \frac{1}{10}s^2}{1 + \frac{5}{4}s + \frac{1}{4}s^2}$$

معادلات المقام أكبر من معادلات البسط

ولذلك يجب أن نقبلها

(حيث تكون معادلات البسط < معادلات المقام)

∴ we use $y(s) = \frac{4 + 5s + s^2}{40 + 28s + 4s^2}$



$$\frac{40 + 28s + 4s^2}{4 + 5s + s^2} \cdot \frac{1}{10} \quad Y_1$$

$$4 + \frac{14}{5}s + \frac{2}{5}s^2$$

$$\frac{\frac{11}{5}s + \frac{3}{5}s^2}{40 + 28s + 4s^2} \cdot \frac{200}{11s} \quad Z$$

$$40 + \frac{120}{11}s$$

$$\frac{\frac{188}{11}s + 4s^2}{\frac{11}{5}s + \frac{3}{5}s^2} \cdot \frac{121}{940} \quad Y$$

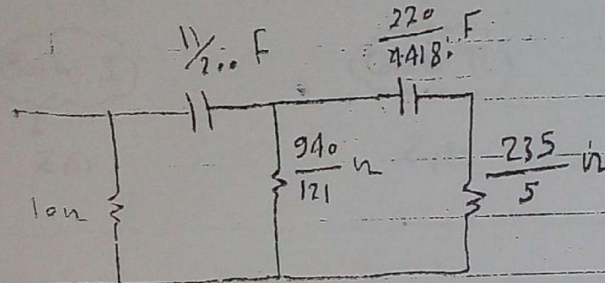
$$\frac{11}{5}s + \frac{121}{235}s^2$$

$$\frac{\frac{20}{235}s^2}{\frac{188}{11}s + 4s^2} \cdot \frac{44180}{220s}$$

$$\frac{188}{11}s$$

$$4s^2 \cdot \frac{20}{235}s^2$$

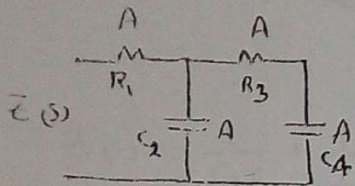
$$\frac{20}{235}s^2$$



5 Four ladder network (Cauer I & Cauer II) دوائر

$$R_1 = R_2 = A \quad \& \quad C_1 = C_2 = A$$

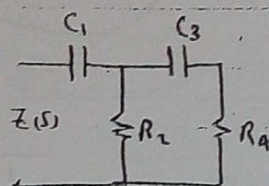
مسألة أربع، إجمالاً، حلها بالخطوات



(1)

Cauer I

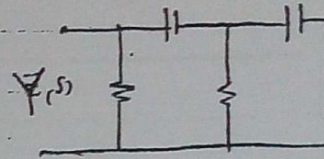
$Z(s)$



(2)

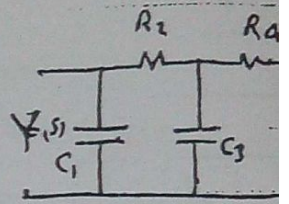
Cauer II

$Z(s)$



(3)

Cauer I



(4)

Cauer II

From (1), Cauer I $\rightarrow$ For $Z(s)$

$$Z(s) = R_1 + \frac{1}{\frac{1}{C_2 s} + \frac{1}{R_3 + \frac{1}{\frac{1}{C_4 s}}}} = A + \frac{1}{A s + \frac{1}{A + \frac{1}{A s}}}$$

$$A + \frac{1}{A s} = \frac{A^2 s + 1}{A s}$$

$$A s + \frac{1}{\frac{A^2 s + 1}{A s}} = A s + \frac{A s}{A^2 s + 1} = \frac{A^3 s^2 + A s + A s}{A^2 s + 1} = \frac{A^3 s^2 + 2 A s}{A^2 s + 1}$$

$$Z(s) = A + \frac{1}{\frac{A^3 s^2 + 2 A s}{A^2 s + 1}} = A + \frac{A^2 s + 1}{A^3 s^2 + 2 A s} = \frac{A^4 s^2 + 2 A^2 s + A^2 s + 1}{A^3 s^2 + 2 A s}$$

$$Z(s) = \frac{A^4 s^2 + 3 A^2 s + 1}{A^3 s^2 + 2 A s}$$

From (2) Case II

$$Z(s) = Z_1 + \frac{1}{Y_2 + \frac{1}{Z_3 + \frac{1}{Y_4}}}$$

← (parallel)
Case I → Case II
For $Z(s)$

$$Z(s) = \frac{1}{C_1 s} + \frac{1}{\frac{1}{R_2} + \frac{1}{\frac{1}{C_2 s} + \frac{1}{R_4}}}$$

$$C_1 = C_2 = A$$

$$R_2 = R_4 = A$$

$$\frac{1}{C_2 s} + R_4$$

$$\frac{1}{R_2} + \frac{1}{\frac{1}{C_2 s} + R_4} = \frac{1}{R_2} + \frac{1}{\frac{1 + C_2 s R_4}{C_2 s A}} = \frac{1}{R_2} + \frac{C_2 s}{1 + C_2 s R_4}$$

$$= \frac{1 + C_2 s R_4 + C_2 s R_2}{R_2 (1 + C_2 s R_4)}$$

$$= \frac{1 + A^2 s + A^2 s}{A(1 + A^2 s)}$$

$$= \frac{1 + 2A^2 s}{A + A^3 s}$$

$$Z(s) = \frac{1}{C_1 s} + \frac{1}{\frac{1 + 2A^2 s}{A + A^3 s}} = \frac{1}{A s} + \frac{A + A^3 s}{1 + 2A^2 s}$$

$$= \frac{1 + 2A^2 s + A^2 s + A^4 s^2}{2A^3 s^2 + A s} = \frac{A^4 s^2 + 3A^2 s + 1}{2A^3 s^2 + A s}$$

Case I & Case II for $Y(s)$

$$Y(s) = Y_1 + \frac{1}{\frac{1}{Y_2} + \frac{1}{Y_3 + \frac{1}{Z_4}}}$$

1 pole/zero
Case I & Case II
For $Y(s)$

let

$$F(s) = K_\infty s + K_0 + \frac{K_1 s}{s + \beta_1} + \dots + \frac{K_i s}{s + \beta_i} \quad \leftarrow \text{For RL network}$$

take $\rightarrow \frac{K_1 s}{s + \beta_1} = Z_1$ For $F(s) = Z(s)$ represent pole

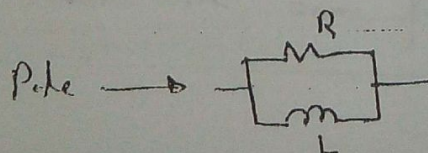
$$\therefore Z_1 = \frac{1}{\frac{s}{K_1} + \frac{\beta_1}{K_1}} = \frac{1}{\frac{1}{K_1} + \frac{1}{\frac{K_1}{\beta_1}} s}$$

$$\therefore Y_1 = \frac{1}{Z_1} = \frac{1}{K_1} + \frac{1}{\frac{K_1}{\beta_1}} s$$

$$\therefore \frac{1}{K_1} \rightarrow \text{represent resistance } (1/R) \Rightarrow R = K_1$$

$$\frac{1}{\frac{K_1}{\beta_1} s} \rightarrow \text{represent inductor } \frac{1}{Ls} \Rightarrow L = \frac{K_1}{\beta_1}$$

So that each pole results in an resistance parallel with inductor, that is if the driving point is impedance



Let $P(s) = Y(s)$

take $\frac{k_1 s}{s + \delta_1} = Y_1$

$$\therefore Y_1 = \frac{1}{\frac{1}{k_1} + \frac{\delta_1}{k_1 s}} = \frac{1}{\frac{1}{k_1} + \frac{1}{\frac{k_1}{\delta_1} s}}$$

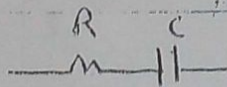
$$\therefore Z_1 = \frac{1}{Y_1} = \frac{1}{k_1} + \frac{1}{\frac{k_1}{\delta_1} s}$$

resistor $\rightarrow$ $R = \frac{1}{k_1}$

capacitor $\rightarrow$ $C = \frac{k_1}{\delta_1}$

Series

Z_1 act as series resistor and capacitor



so it can be synthesized as RC circuit

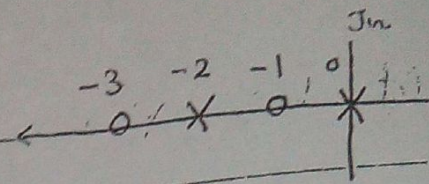
For function $F(s)$ that achieves the RL properties

if $F(s) = Z(s)$ it is RL network

if $F(s) = Y(s)$ it is RC network

and they have the same form.

$$(b) F(s) = \frac{(s+1)(s+3)}{s(s+2)}$$



RC دوائر

(Foster I)

$$F(s) = K_0 + \frac{K_1}{s} + \frac{K_2}{s+2} = \frac{(s+1)(s+3)}{s(s+2)}$$

$$K_0 = \lim_{s \rightarrow \infty} F(s) = [1]$$

$$K_0 = \lim_{s \rightarrow \infty} s F(s) = \lim_{s \rightarrow \infty} \frac{(s+1)(s+3)}{(s+2)} = \frac{3}{2}$$

$$K_1 = \lim_{s \rightarrow -2} (s+2) F(s) = \lim_{s \rightarrow -2} \frac{(s+1)(s+3)}{s} = \frac{(-1)(1)}{-2} = \frac{1}{2}$$

$$F(s) = 1 + \frac{3/2}{s} + \frac{1/2}{s+2}$$

$$F(s) = 1 + \frac{1}{\frac{2}{3}s} + \frac{1}{2s+4}$$

$F(s) = Z(s) \Rightarrow$ it is Foster I

$$Z(s) = 1 + \frac{1}{\frac{2}{3}s} + \frac{1}{2s+4}$$

$$Y_1 = 2s + 4$$

قوة
توازي
سلسلة

